ENGINEERING PHYSICS

FOR DIPLOMA STUDENTS

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UNIT – 1

UNITS AND DIMENSIONS

PHYSICAL QUANTITIES -:

- All the quantities in terms of which laws of physics are described & whose measurement is necessary are physical quantities.

OR

- A quantity that can be measured is called as physical quantity.
- It is also known as measurable quantity.
- Examples – length, mass, velocity, area, volume, density, temperature etc.
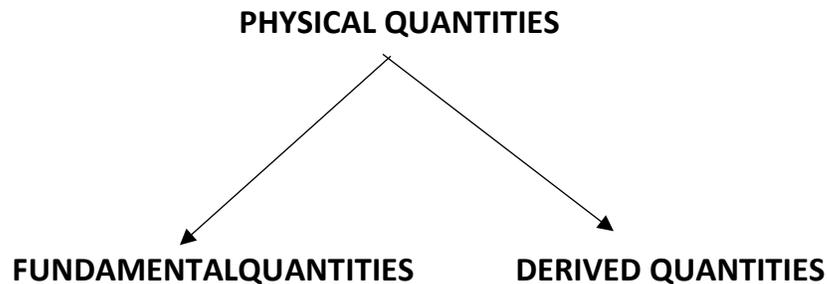
NON- PHYSICAL QUANTITIES

- A quantity that cannot be measured is called as non-physical quantity.
- It is also known as non-measurable quantity.
- Examples -: building, bus, road, t.v., radio etc.

DEFINITION OF SOME PHYSICAL QUANTITIES -:

1. MASS – Mass of a body is defined as the quantity of matter in the body, which can never be zero.

2. LENGTH – Length of an object may be defined as the distance of separation between any two points at the extreme end of the object.

3. TIME – It is very difficult to define time precisely. According to Einstein, "Time is what a clock reads".

4. DISTANCE – Distance of an object is defined as the total or complete path or actual path travelled by an object.

5. DISPLACEMENT – Displacement of a body is defined as the shortest distance between the final position & the initial position of the motion of the body.

6. SPEED – Speed of a body is defined as the distance covered by the body in one second.

7. VELOCITY – Velocity of a body is defined as the time rate of change of displacement.

8. ACCELERATION - Acceleration of a body is defined as the rate of change of velocity.

9. FORCE – (Qualitatively) Force is defined as that pull or push which produces or tends to produce, destroys or tends to destroy motion in a body, increases or decreases the speed of the body or changes its direction of motion.

10. MOMENTUM – (Qualitatively) Momentum of a body is defined as the amount of motion contained in a body.

11. WORK – (Qualitatively) Work is said to be done if a force acting on a body, displaces a body through a certain distance & the force has some component along the displacement.

12. POWER – The rate at which work is done is called power.

13. ENERGY – It is ability of the body to do some work.

CLASSIFICATION OF PHYSICAL QUANTITIES

PHYSICAL QUANTITIES

FUNDAMENTALQUANTITIES   **DERIVED QUANTITIES**

FUNDAMENTAL QUANTITIES -:

- A physical quantity which does not depend on any other physical quantity for its measurement is known as fundamental (Basic) quantity.

OR

- Those quantities which can neither be derived from one another nor they can be resolved (broken) into anything more basic are called fundamental quantities.
- There are 7 fundamental quantities.
- Examples- mass, length, time, temperature, current, luminous intensity, amount of substance.

DERIVED QUANTITIES -:

- The physical quantities which are expressed in terms of more than one fundamental quantities are called as derived quantities.

OR

- The physical quantity that depends upon other physical quantities for its measurement is known as derived quantity.
- Except 7 fundamental quantities, all other quantities are derived quantities.
- Examples -: area, volume, force, momentum etc.

SUPPLEMENTARY QUANTITIES -:

- Plane angle (θ)
- Solid angle (Ω)

UNITS -:

- The chosen standard of measurement of a quantity which has essentially the same nature as that of the quantity is called the unit of the quantity.

OR

- Internationally accepted reference standard to measure a physical quantity.

CHARACTERISTICS OF A UNIT -:

- It should be invariable.
- It should be of convenient size.
- It should be easily available.
- It should be non- perishable.

SYSTEMS OF UNITS -:

There are 3 major systems of units.

1. F.P.S. SYSTEM -:

- In this system the basic units of length, force (instead of mass) & time chosen as the fundamental quantities are foot, pound & second respectively.
- It is also known as British system.

2. CGS SYSTEM -:

- In this system the basic units of length, mass & time chosen as the fundamental quantities are centimeter, gram & second respectively.
- It is also known as French or Gaussian system.

3. MKS SYSTEM -:

- In this system the basic units of length, mass & time chosen as the fundamental quantities are meter, kilogram & second respectively.
- It is also known as Metric system.

S.I. UNITS -: (THE INTERNATIONAL SYSTEM OF UNITS)

Each of above 3 systems of units is sufficient only to describe all physical quantities in mechanics. However for the description of physical quantities in field of electricity, thermodynamics & optics introduction of some additional basic units is necessary. Therefore the systems of units mentioned above are not complete & coherent.

- The 11th **General Conference On Weights & Measures** which met in Paris in 1960, introduced a new system of units called S.I. System, which covers all fields of science & technology.
- The S.I. system consist of 3 classes of units.

1. Base units – The units of fundamental quantities are called Base units or fundamental units.

QUANTITY	UNIT	SYMBOL
Length(l)	meter	m
Mass(m)	kilogram	kg
Time(t)	second	s

Electric current(i)	ampere	A
Temperature(T)	kelvin	K
Luminosity	candela	Cd
Amount of substance	mole	mol

2. Supplementary units – The units of Supplementary quantities are called Supplementary units.

QUANTITY	UNIT	SYMBOL
Plane angle	Radian	rad
Solid angle	steradian	sr

3. Derived units – The units of derived quantities are called derived units.

Derived units can be obtained by simple multiplication & or division of base and supplementary units.

Examples -

QUANTITY	UNIT	SYMBOL
Area	(meter)^2	m^2
momentum	(kilogram.meter)/second	kgm s^{-1}

PREFIXES FOR LARGER & SMALLER UNITS IN POWERS OF TEN

SUB MULTIPLES (SMALLER)			MULTIPLES (LARGER)		
PREFIX	ABBREVIATION	VALUE	PREFIX	ABBREVIATION	VALUE
deci-	d	10^{-1}	*Deca-*	D	10^1
centi-	c	10^{-2}	*Hecta-*	H	10^2
milli-	m	10^{-3}	*Kilo-*	k	10^3
micro-	μ	10^{-6}	*Mega-*	M	10^6
nano-	n	10^{-9}	*Giga-*	G	10^9
pico-	p	10^{-12}	*Tera-*	T	10^{12}
femto-	f	10^{-15}	*Peta-*	P	10^{15}
atto-	a	10^{-18}	*Exa-*	E	10^{18}

DIMENSIONS -:

- Dimensions of a physical quantity are the powers to which the fundamental units be raised in order to represent that quantity.

DIMENSIONAL FORMULA -:

- Dimensional formula of a physical quantity is the formula which tells us how and which of the fundamental units have been used for the measurement of that quantity.

QUANTITY	DIMENSION
Mass	$[M^1]$ or $[M^1L^0T^0]$
Length	$[L^1]$ or $[M^0L^1T^0]$
Time	$[T^1]$ or $[M^0L^0T^1]$
Temperature	$[K^1]$ or $[M^0L^0T^0K^1A^0]$
Current	$[A^1]$ or $[M^0L^0T^0K^0A^1]$

- The pure numbers (1, 2, 3...π, e etc) & all trigonometric functions have no dimensions.

DETERMINATION OF DIMENSIONS OF A PHYSICAL QUANTITY -:

Dimensions of a physical quantity can be determined as follows –

1. Write the formula for the quantity, with the quantity of L.H.S. of the equation.

2. Convert all the quantities on R.H.S. into the fundamental quantities mass, length & time.

3. Substitute M, L & T for mass, length & time respectively.

4. Collect terms of M, L & T in a square bracket without commas in between them & find their resultant powers which give the dimensions of the quantity in mass, length & time respectively.

Examples-

1. Volume = length $\times$ breadth $\times$ thickness

$$= [L] \times [L] \times [L]$$
$$= [L^3] \text{ or } [M^0L^3T^0]$$

So, the dimensions of volume are 0, 3 & 0 in mass, length & time respectively.

2. Velocity $= \dfrac{\text{Displacement}}{\text{Time}}$

$$= \dfrac{[L^1]}{[T^1]}$$
$$= [L^1T^{-1}]$$

So, the dimensions of velocity are 0, 1 & -1 in mass, length & time respectively.

DIMENSIONAL EQUATION -:

- An equation written in the following manner is called dimensional equation.

$$Volume = [M^0L^3T^0]$$

PRINCIPLE OF HOMOGENEITY -:

- It states that the dimensional formula of every term on the two sides of a correct relation must be same.
- Two quantities in addition or subtraction should have same dimension.
- Quantities on either side of an expression or equation should have the same dimension.
- Using this principle we can check the correctness of a physical equation.

Uses of dimensional analysis -:

1. To convert the values of a physical quantity from one system to another.

2. To check the correctness of a given relation.

3. To derive a relation between various physical quantities.

All these uses are based upon the "**Principle of Homogeneity**".

CHECKING THE DIMENSIONAL CORRECTNESS OF PHYSICAL RELATIONS -:

Let us check the correctness of the relation

$$S = ut + \frac{1}{2}at^2$$

L.H.S.

Dimensional formula of **S**   or $[S] = [L^1]$

R.H.S.

Dimensional formula of **ut**   or $[ut] = [u][t]$
$$= [L^1T^{-1}][T^1]$$
$$= [L^1]$$

Dimensional formula of $\frac{1}{2}at^2$ or $[\frac{1}{2}at^2] = [a][t^2]$
$$= [L^1T^{-2}][T^2]$$
$$= [L^1]$$

So L.H.S. = R.H.S.

Since the dimensional formulae of all the terms involved in the above relation are $[L^1]$.

So the given relation is **dimensional correct**.

UNIT-2

SCALARS AND VECTORS

- Physical quantities can be classified into the following 2 categories depending upon their magnitude & direction.

SCALAR QUANTITIES:-

- Scalar quantities are those quantities which require only the magnitude for their complete specification.
- Ex- mass, length, volume, density, temperature, electric charge, energy, work etc.

VECTOR QUANTITIES:-

- Vector quantities are those quantities which require magnitude as well as direction for their complete specification & they obey vector algebra.
- They cannot be added algebraically as that of scalar quantities.
- Ex- displacement, velocity, acceleration, force, momentum, weight etc.

REPRESENTATION OF A VECTOR:-

A vector can be represented by the following steps.

1. Draw a line parallel to the direction of the vector.

2. Cut a length of the line so that it represents the magnitude of the vector on a certain convenient scale.

3. Put an arrowhead in the direction of the vector.

This line with the arrowhead, represents the given vector

A vector is written with an arrowhead over its symbol like '$\vec{A}$' .

O ————————————— v = 40m/s ————————————→ P

←————————————— 4cm —————————————→
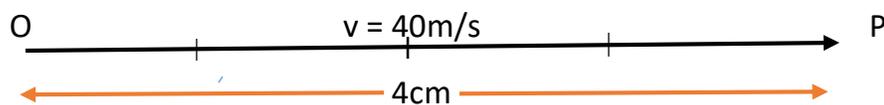
(Representation of velocity vector)

- ✓ Length of the vector represents magnitude ($|\vec{A}|$).
- ✓ Arrowhead represents the direction with the help of an angle with horizontal or vertical.

Head

Magnitude = $|\vec{A}|$

$\vec{A}$

θ

Tail
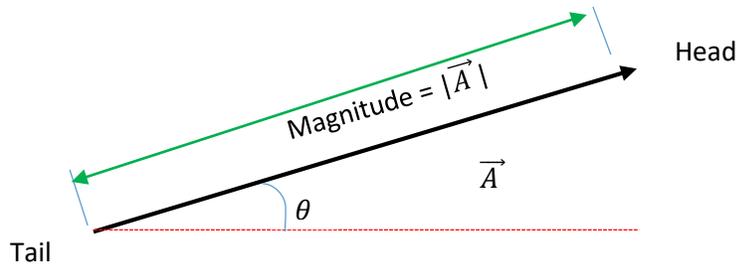
TYPES OF VECTORS:-

NULL VECTOR – It is a vector having zero magnitude & arbitrary direction.

EQUAL VECTORS – Two vectors are said to be equal if they possess the same magnitude & direction.

$\vec{u}$

$\vec{v}$
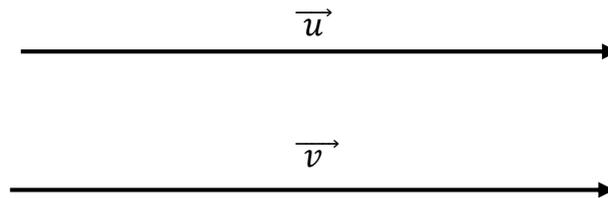
NEGATIVE VECTOR – A vector is said to be a negative vector of another one, if it is represented by a line having same length as that of the second & is directed in opposite direction.
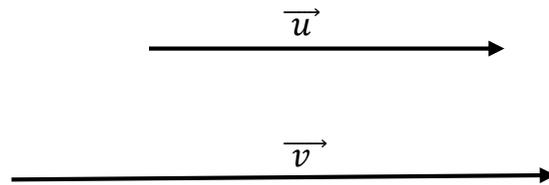
$\vec{u}$

$-\vec{u}$
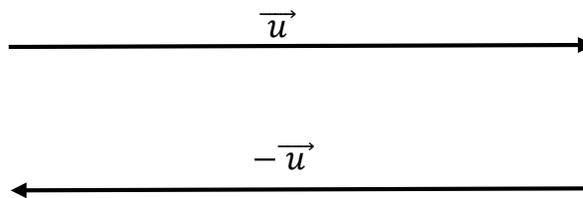
CO-INITIAL VECTOR – A number of vectors having a common initial point are called co-initial vectors.

$\vec{A}$

$\vec{B}$

$\vec{C}$

O

$\vec{D}$
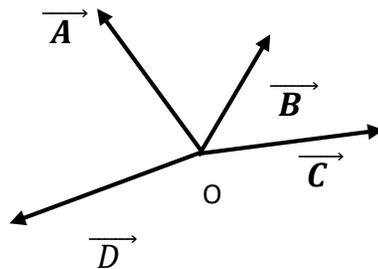
COLINEAR VECTOR – Vectors having a common line of action are called colinear vectors. There are rwo types of colinear vectors .
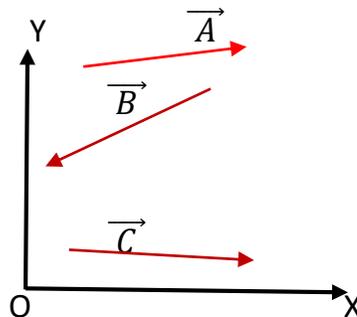
1. Parallel vector ($\theta = 0°$) – Two vectors (which may have different magnitudes) acting along same direction are called parallel vectors.

$$\overrightarrow{u}$$

$$\overrightarrow{v}$$

2. Anti-parallel vector ($\theta = 180°$) – Two vectors which are directed in opposite direction are called anti-parallel vectors.

$$\overrightarrow{u}$$

$$\overrightarrow{v}$$
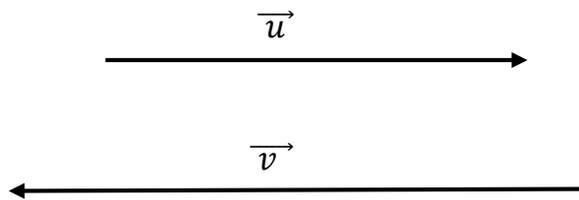
CO-PLANAR VECTORS – Vectors situated in one plane irrespective of their directions are known as coplaner vectors.

LOCALISED VECTORS – Vector whose initial point (tail) is fixed is said to be a localised or a fixed vector.

Ex- position vector can be considered as a localisesd vector because position vector of every point starts from origin.

NON-LOCALISED VECTORS – Vector whose initial point (tail) is not fixed is said to be a non-localised or a free vector.

Ex- Vectors representing force, momentum etc.

TRIANGLE'S LAW OF VECTOR ADDITION :-

- It is a law for the addition of two vectors.

Statement - " If two vectors are represented in magnitude & direction by the two sides of a triangle, taken in the same order, then their resultant is represented in magnitude & direction by the third side of the triangle taken in opposite order."
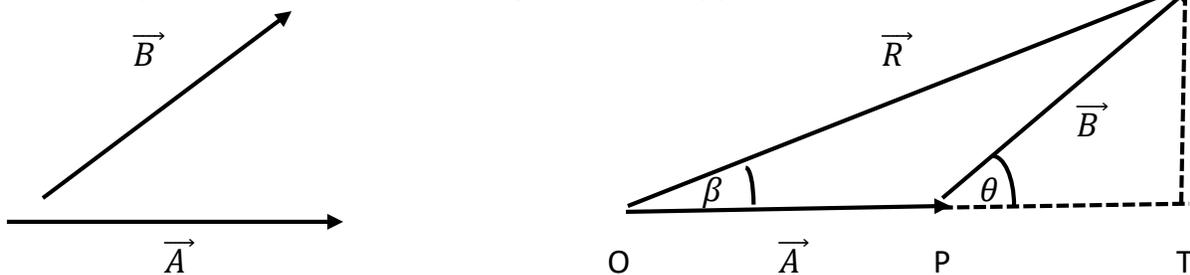
Let the two vectors $\vec{A}$ & $\vec{B}$ acting at a point be represented by the two sides OP & PQ of the Δ OPQ, taken in same order. 'θ' is the angle between $\vec{A}$ & $\vec{B}$.

According to the triangle's law, the third side OQ of the triangle taken in opposite order gives the resultant $(\vec{R})$.

$$\boxed{\vec{R} = \vec{A} + \vec{B}}$$

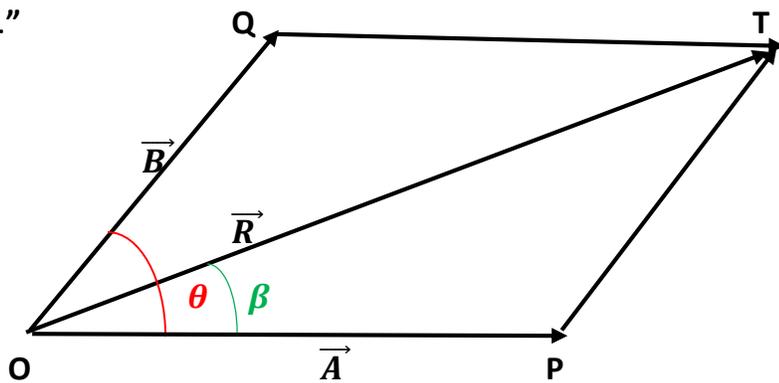The magnitude of the resultant vector is given by

$$\boxed{\left|\vec{R}\right| = \sqrt{A^2 + B^2 + 2AB \; \cos\theta}}$$

& the direction of resultant vector is given by

$$\boxed{\beta = \tan^{-1}\frac{B\sin\theta}{A + B\cos\theta}}$$

PARALLELOGRAM LAW OF VECTOR ADDITION -:

Statement – " It states that if two vectors acting simultaneously at a point are represented in magnitude & direction by the two sides of a parallelogram drawn from a point, their resultant given in magnitude & direction by the diagonal of the parallelogram passing through that point."

Let the two vectors $\vec{A}$ & $\vec{B}$ acting simultaneously at a point 'O' be represented by the two adjacent sides OP & OQ of a parallelogram OPTQ. 'θ' is the angle between $\vec{A}$ & $\vec{B}$.

According to the parallelogram law of vectors, the diagonal OT gives the resultant ($\vec{R}$) of $\vec{A}$ & $\vec{B}$.

$$\boxed{\vec{R} = \vec{A} + \vec{B}}$$

The magnitude of the resultant vector is given by

$$\boxed{|\vec{R}| = \sqrt{A^2 + B^2 + 2AB \, \cos\theta}}$$

& the direction of resultant vector is given by

$$\boxed{\beta = \tan^{-1}\frac{B\sin\theta}{A + B\cos\theta}}$$

UNIT VECTOR – Unit vector of any vector is a vector having a unit magnitude, drawn in the direction of the given vector.

It represents the direction of a given vector.

$\vec{A}$ can be written as $\qquad\qquad \boxed{\vec{A} = |\vec{A}| \, \hat{A}}$

$|\vec{A}|$ = magnitude of $\vec{A}$
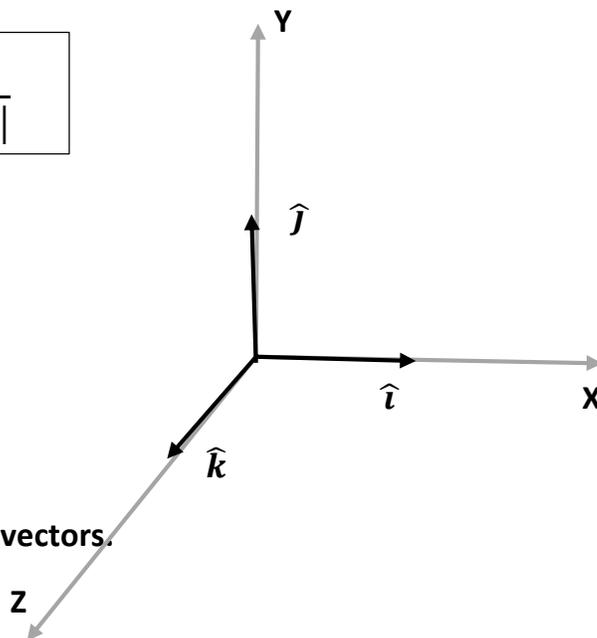
$\hat{A}$ = Unit vector of $\vec{A}$

$$\boxed{\hat{A} = \frac{\vec{A}}{|\vec{A}|}}$$

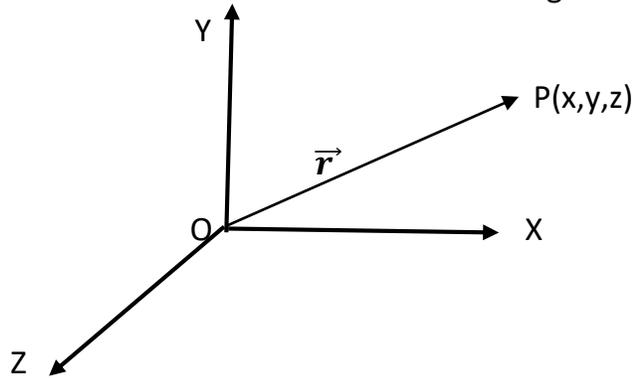$\hat{\imath}$ - unit vector along X-axis

$\hat{\jmath}$ - unit vector along Y-axis

$\hat{k}$ - unit vector along Z-axis

$\hat{\imath}, \hat{\jmath}$ & $\hat{k}$ are called orthogonal unit vectors.
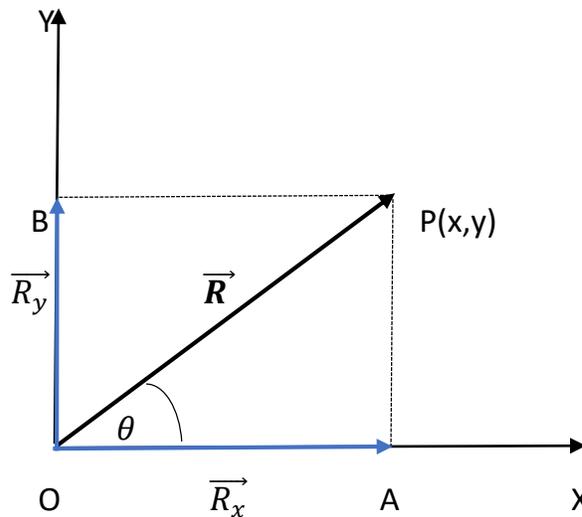
POSITION VECTOR – Position vector of any point, with respect to an arbitrarily chosen origin is defined as the vector which connects the origin & the point & is directed towards the point.

'$\vec{r}$' is the position vector of the point 'P'

RESOLUTION OF VECTORS :-

- Resolution of vectors is the process of obtaining the component vectors which when combined, according to laws of vector addition, produce the given vector.
- **Rectangular components** of a given vector are its components in two mutually perpendicular directions in the plane of the given vector.

Let $\overrightarrow{OP}$ ($\vec{R}$) be the position vector of a point P(x,y). From P draw PA & PB perpendiculars on X-axis & Y-axis respectively.

$\overrightarrow{OA}$ & $\overrightarrow{OB}$ are called x-component & y-component of $\vec{R}$ respectively.

Let OA = R_x & OB = R_y. θ be the angle which the given vector $\vec{R}$ makes with X-axis.

In $\Delta\ OAP$, $\cos\theta = \dfrac{OA}{OP}$

$OA = OP\ \cos\theta$

$$\boxed{R_x\ =\ R\ \cos\theta}\ \ \Longrightarrow\ \textbf{Horizontal component}$$

In $\Delta\ OBP$, $\sin\theta = \dfrac{OB}{OP}$

$OB = OP\ \sin\theta$

$$\boxed{R_y\ =\ R\ \sin\theta}\ \ \Longrightarrow\ \textbf{Vertical component}$$

- R_x & R_y may not be termed as the magnitudes of $\vec{R_x}$ & $\vec{R_y}$ respectively.
- This is because the magnitude of a vector is always positive whereas R_x & R_y may have positive, negative or zero value depending upon the value of θ. Therefore R_x & R_y are called **scalar rectangular components of $\vec{R}$.**

VECTOR MULTIPLICATION -:

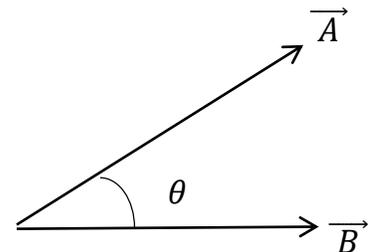There are two ways in which two vectors can be multiplied together.

DOT PRODUCT -:

- Dot product of two vectors is defined as the product of their magnitudes & the cosine of the smaller angle between the two. It is written by putting a dot **(.)** between two vectors .
- Let $\vec{A}$ & $\vec{B}$ be the two vectors drawn from a point 'O' & 'θ' be the angle between them.

Then dot product of $\vec{A}$ & $\vec{B}$ is given by

$$\boxed{\vec{A}.\vec{B} = AB\ \cos\theta}$$
$$= A(B\ \cos\theta)$$

$$= A(componet\ of\ \vec{B}\ in\ the\ direction\ of\ \vec{A}\ )$$

- So, the dot product of two vectors can also be defined as the product of magnitude of one vector & the component of the other vector in the direction of the first.
- The result of this product does not possess any direction. Hence it is also called as **Scalar Product**.

Since, $\hat{\imath}, \hat{\jmath}$ & $\hat{k}$ are mutually perpendicular to each other, so

$$\boxed{\begin{array}{c} \hat{\imath} \cdot \hat{\jmath} = \hat{\jmath} \cdot \hat{k} = \hat{k} \cdot \hat{\imath} = 0 \\ \hat{\imath} \cdot \hat{\imath} = \hat{\jmath} \cdot \hat{\jmath} = \hat{k} \cdot \hat{k} = 1 \end{array}}$$

DOT PRODUCT IN TERMS OF RECTANGULAR COMPONENTS -:

Let, $\vec{A} = A_x \hat{\imath} + A_y \hat{\jmath} + A_z \hat{k}$

&

$\vec{B} = B_x \hat{\imath} + B_y \hat{\jmath} + B_z \hat{k}$

A_x, A_y, A_z are rectangular components of $\vec{A}$ & B_x, B_y, B_z are rectangular components of $\vec{B}$.

Then $$\boxed{\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z}$$
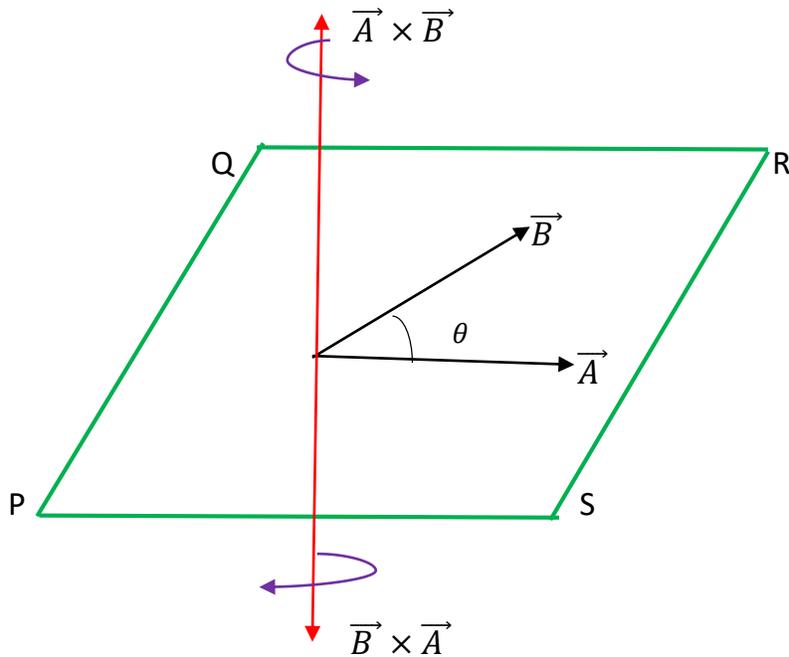
CROSS PRODUCT -:

- Cross product of two vectors $\vec{A}$ & $\vec{B}$ is defined as a single vector $\vec{C}$, whose magnitude is equal to the product of their individual magnitudes & the sine of the smaller angle between them & is directed along the normal to the plane containing $\vec{A}$ & $\vec{B}$.

- It is written by putting a cross **(×)** between the vectors.

$$\boxed{\vec{A} \times \vec{B} = \vec{C} = AB \, \sin\theta \, \hat{n}}$$

- Where $\hat{n}$ is the unit vector in a direction perpendicular to the plane containing $\vec{A}$ & $\vec{B}$.

- The direction of $\vec{C}$ be determined by applying **Right Hand Thumb Rule**.

- It is also known as **Vector Product**.

RIGHT HAND THUMB RULE -:

- Imagine the normal to the plane PQRS containing $\vec{A}$ & $\vec{B}$ to be held in the right hand with the thumb erect. If the fingers curl in the direction from $\vec{A}$ to $\vec{B}$ then the direction of thumb gives the direction of $\vec{A} \times \vec{B}$

Since $\hat{\imath}, \hat{\jmath}$ & $\hat{k}$ are mutually perpendicular to each other, so

$$|\hat{\imath} \times \hat{\imath}| = |\hat{\jmath} \times \hat{\jmath}| = |\hat{k} \times \hat{k}| = 0$$

$$\hat{\imath} \times \hat{\jmath} = \hat{k} \qquad\qquad \hat{\jmath} \times \hat{\imath} = -\hat{k}$$
$$\hat{\jmath} \times \hat{k} = \hat{\imath} \qquad\qquad \hat{k} \times \hat{\jmath} = -\hat{\imath}$$
$$\hat{k} \times \hat{\imath} = \hat{\jmath} \qquad\qquad \hat{\imath} \times \hat{k} = -\hat{\jmath}$$

CROSS PRODUCT IN TERMS OF RECTANGULAR COMPONENTS -:

Let, $\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$

&

$\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$

A_x, A_y, A_z are rectangular components of $\overrightarrow{A}$ & B_x, B_y, B_z are rectangular components of $\overrightarrow{B}$.

Then

$$\overrightarrow{A} \times \overrightarrow{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\overrightarrow{A} \times \overrightarrow{B} = \left(A_y B_z - A_z B_y\right)\hat{\imath} - \left(A_x B_z - A_z B_x\right)\hat{\jmath} + \left(A_x B_y - A_y B_x\right)\hat{k}$$

UNIT- 3
KINEMATICS

REST -:
- A body is said to be at rest if it does not change its position with respect to its surroundings.

MOTION -:
- A body is said to be in motion if it changes its position with respect to the surroundings.

DISPLACEMENT-:
- Displacement of a body is a vector connecting the initial & final positions of the body & is directed away from initial towards position irrespective of path followed by the body.
- Dimension is $[L^1]$
- S.I. unit is meter (m).

SPEED -:
- Speed of a body is defined as the distance covered by the body in one second.
- If Δs is the distance covered by a body in time Δt, then

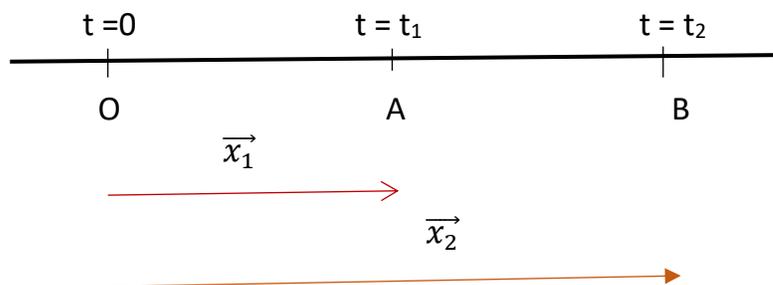$$\text{Average speed } = \frac{\Delta s}{\Delta t}$$

- If the time interval Δt is chosen to be very small i.e. $\Delta t \to 0$, the corresponding speed is called instantaneous speed.

$$\text{Instantaneous speed } = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

- Speed is a scalar quantity.
- Dimensional formula - $[M^0L^1T^{-1}]$.
- S.I. unit is meter per second (m/s).

VELOCITY -:
- Velocity of a body is defined as the rate of change of displacement.

- Consider a body starting from O at t = 0 & reaching A at time t = t_1 & B at time t = t_2, such that its displacement at these instants are $\vec{x_1}$ & $\vec{x_2}$ respectively.

- Average velocity, $\quad \vec{v}_{avg} = \frac{\vec{x_2} - \vec{x_1}}{t_2 - t_1} = \frac{\Delta \vec{x}}{\Delta t}$

- If the time interval Δt is very small i.e. B lies very close to A, then the corresponding value of velocity is called instantaneous velocity.
- Instantaneous velocity, $\quad \vec{v}_{inst} = \lim\limits_{\Delta t \to 0} \dfrac{\Delta \vec{x}}{\Delta t} = \dfrac{d\vec{x}}{dt}$
- Since displacement is a vector quantity, velocity is also a vector quantity.
- Dimensional formula - $[M^0L^1T^{-1}]$.
- S.I. unit is meter per second (m/s).

There are two types of velocities.

   1. UNIFORM VELOCITY -: Velocity of a body is said to be uniform if it covers equal displacements in equal intervals of time.

- In this case we find average velocity is given by

$$\vec{v}_{avg} = \frac{\Delta \vec{x}}{\Delta t}$$

   2. Non-uniform velocity -: velocity of body is said to be non-uniform if it covers unequal displacements in equal intervals of time.

- In this case we find instantaneous velocity is given by

$$\vec{v}_{inst} = \lim\limits_{\Delta t \to 0} \frac{\Delta \vec{x}}{\Delta t} = \frac{d\vec{x}}{dt}$$

ACCELERATION -:

- Acceleration is defined as the change in velocity in one second.

<div align="center">OR</div>

- Acceleration is defined as the rate of change of velocity.
- Acceleration is a vector quantity. Direction of acceleration is along the direction in which change in velocity is represented.
- Dimensional formula - $[M^0L^1T^{-2}]$.
- S.I. unit is meter per (second)^2 or (m/s^2).

There are two types of accelerations.

   1. Uniform acceleration -: The acceleration of body is said to be uniform if its velocity changes by equal amount in in equal intervals of time.

- In this case we find the average acceleration of the body.

$$\vec{a}_{avg} = \frac{\vec{v_2} - \vec{v_1}}{t_2 - t_1} = \frac{\Delta \vec{v}}{\Delta t}$$

   2. Non-uniform acceleration -: The acceleration of a body is said to be non-uniform if its velocity changes by unequal amounts in equal intervals of time.

- In this case we find instantaneous acceleration of the body is given by

$$\vec{a}_{inst} = \lim\limits_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

FORCE -:

- *(Qualitatively)* Force is defined as that pull or push which produces or tends to produce, destroys or tends to destroy motion in a body, increases or decreases the speed of the body or changes its direction of motion.
- *(Quantitatively)* The rate of change of momentum of a body is directly proportional to the impressed (applied) force & takes place in the direction of the force.
- Newton's first law provides a qualitative definition of the force while second law provides the quantitative definition of force.
- From Newton's second law $Force = mass \times acceleration$

$$\vec{F} = m\vec{a}$$

- Force is a vector quantity.
- Dimensional formula - $[M^1L^1T^{-2}]$.
- S.I. unit is newton (N).

There are two types of units in which force is measured.

- **Absolute unit -:** Absolute unit of force is the force which produces unit acceleration in a body of unit mass.

  In C.G.S. system – 1 dyne.

$$F = ma$$

If m = 1 g, a = 1 cm/s² then $F = 1\,{gcm}/{s^2} = 1\,dyne$

  In S.I. – 1 newton (N)

$$F = ma$$

If m = 1 kg, a = 1 m/s² then $F = 1\,{kgm}/{s^2} = 1\,N$

  1 N = 10⁵ dyne

- **Gravitational unit -:** Gravitational unit of force is the force which produces an acceleration equal to the acceleration due to gravity in a unit mass.

  In C.G.S. system – 1 gram weight or gram force.

$$F = ma$$

If m = 1g, a = 980 cm/s² then $F = 980\,{gcm}/{s^2} = 1\,g\,wt\;or\;1\,g\,f$

  In S.I. – 1 kilogram weight or kilogram force

$$F = ma$$

If m = 1 kg, a = 9.8 m/s² then $F = 9.8\,{kgm}/{s^2} = 1\,kg\,wt\;or\;1\,kg\,f$

$$1\,kg\,wt = 9.8\,N$$
$$1\,g\,wt = 980\,dyn$$

EQUATIONS OF MOTION UNDER GRAVITY -:

- Let $\vec{u} = initial\ velocity\ of\ the\ body$
  $$\vec{v} = final\ velocity\ of\ the\ body$$
  $$\vec{a} = acceleration\ produced\ in\ the\ body$$
  $$\vec{s} = displacement\ of\ the\ body$$
  $$t = time\ interval$$
- Equations involving $\vec{u}, \vec{v}, \vec{a}, \vec{s}$ & t are called equations of motion.
  $$\vec{v} = \vec{u} + \vec{a}t$$
  $$\vec{s} = \vec{u}t + {}^1\!/_2\,\vec{a}t^2$$
  $$v^2 - u^2 = 2as$$

1. FOR UPWARD MOTION –

$a = -g\ (g\ is\ the\ acceleration\ due\ to\ gravity)$
Distance travelled by the body in t second = h
$$v = u - gt$$
$$h = ut - {}^1\!/_2\,gt^2$$
$$v^2 - u^2 = -2gh$$

2. FOR DOWNWARD MOTION –

$a = +g\ (g\ is\ the\ acceleration\ due\ to\ gravity)$
Distance travelled by the body in t second = h
$$v = u + gt$$
$$h = ut + {}^1\!/_2\,gt^2$$
$$v^2 - u^2 = 2gh$$

CIRCULAR MOTION -:

- A body is said to move in circular motion if it moves in such a way that its distance from a fixed point always remains constant.
- The path of body is called a circle. Fixed point is called center of the circle & the fixed distance is called the radius of the circle.

ANGULAR DISPLACEMENT -:

- Angular displacement of a particle undergoing rotational motion is defined as the angle turned by its radius vector.
- It is a vector quantity. It is directed along the axis of rotation. Its direction can be determined by using right hand thumb rule.
- For **anti-clockwise rotation** the direction of angular displacement is **upward**. For **clockwise rotation** the direction of angular displacement is **downward**.

RELATION BETWEEN LINEAR DISPLACEMENT & ANGULAR DISPLACEMENT
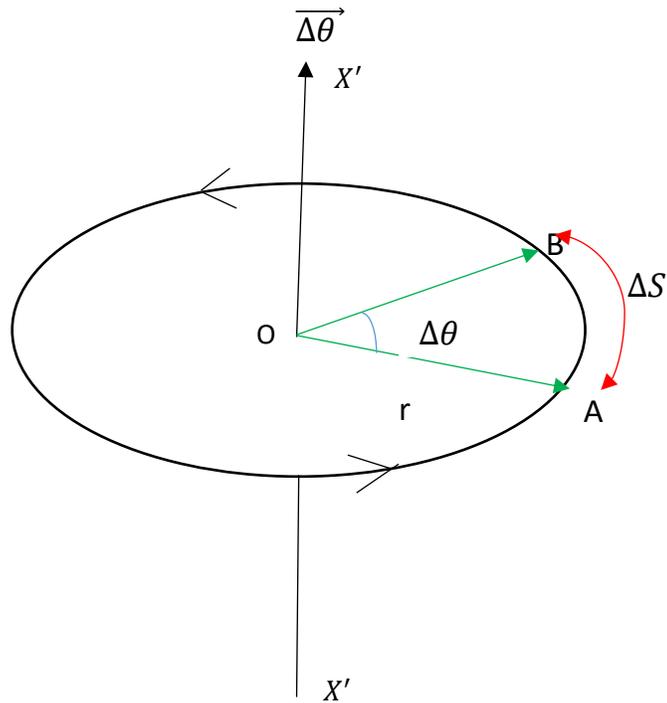
- Suppose a body undergoes uniform circular motion (motion in a circle with uniform speed) about the axis XX$'$. Let ΔS be the linear displacement corresponding to the angular displacement $\Delta\theta$ as shown in the figure.

Then $\qquad\qquad\qquad \Delta\theta = \dfrac{\Delta S}{r}$

$\Rightarrow \qquad \boxed{\Delta S = r\Delta\boldsymbol{\theta}}$

$$\boxed{\boldsymbol{linear\ displacement = radius\ \times\ angular\ displacement}}$$

- In **vector form** the above equation can be written as

$$\boxed{\Delta\vec{\boldsymbol{S}} = \Delta\vec{\boldsymbol{\theta}} \times \vec{\boldsymbol{r}}}$$

- Angular displacement is a dimensionless quantity.
- S.I. unit – radian.

ANGULAR VELOCITY -:

- Angular velocity of a particle undergoing rotational motion is defined as the rate of change of angular displacement with time.

- It is a vector quantity & is represented along the axis of rotation according to right hand thumb rule.
- Suppose a particle undergoing uniform circular motion about the axis XX'
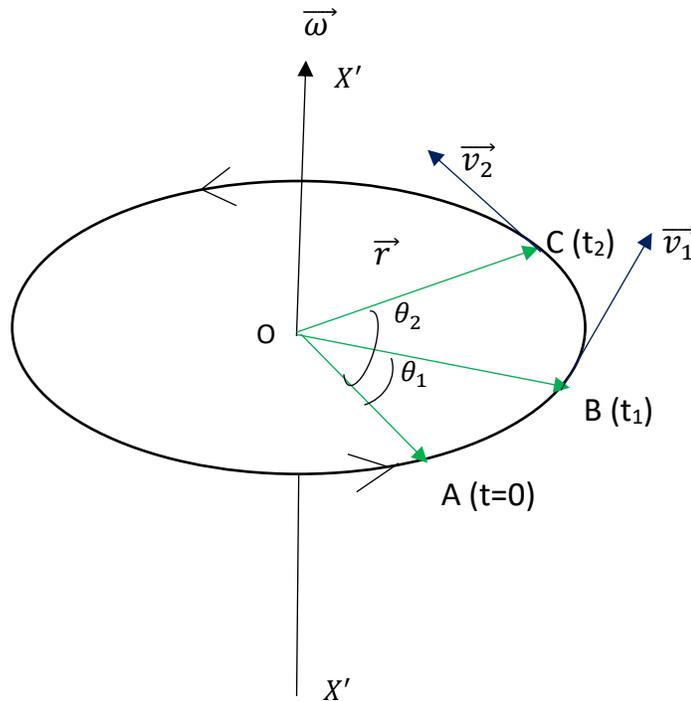
- Let the particle moves from A to B in a time t_1 & to C in a time t_2.
- If $\overrightarrow{\theta_1}$ & $\overrightarrow{\theta_2}$ are the corresponding angular displacements at these instants then the average angular velocity ($\overrightarrow{\omega}_{avg}$) is given by

$$\overrightarrow{\omega}_{avg} = \frac{change\ in\ angular\ displacement}{change\ in\ time}$$

$$\overrightarrow{\omega}_{avg} = \frac{\overrightarrow{\theta_2} - \overrightarrow{\theta_1}}{t_2 - t_1} = \frac{\Delta\overrightarrow{\theta}}{\Delta t}$$

- Instantaneous angular velocity $\overrightarrow{\omega}_{inst} = \frac{d\overrightarrow{\theta}}{dt}$
- Dimensional formula = $[M^0L^0T^{-1}]$
- S.I. unit = radian/second.

RELATION BETWEEN LINEAR VELOCITY & ANGULAR VELOCITY -:

- Suppose the body moves along the circular path with uniform linear velocity 'v'.

  Then from the figure $\quad v = \frac{AB}{t_1}$

$$\Rightarrow \quad AB = vt_1 \text{---------------- (1)}$$

  & also $\quad\quad \theta_1 = \frac{AB}{r}$

$$\Rightarrow \quad AB = \theta_1 r \text{-------------------- (2)}$$

From equ. (1) & (2)

$$vt_1 = \theta_1 r$$

$$\Rightarrow \quad v = \frac{\theta_1}{t_1} r$$

$$\Rightarrow \quad v = \omega r$$

$$\boxed{linear\ velocity = radius\ \times angular\ velocity}$$

- Since $\vec{r}, \vec{\omega}$ & $\vec{v}$ are all vector quantities. So the above equation in **vector form** can be written as

$$\boxed{\vec{v} = \vec{\omega} \times \vec{r}}$$

ANGULAR ACCELERATION -:

- Angular acceleration of a body is defined as the time rate of change of angular velocity.
- Suppose $\vec{\omega}_1$ & $\vec{\omega}_2$ are the angular velocities of the particle at the instants t_1 & t_2.

  Then average angular acceleration ($\vec{\alpha}_{avg}$) is given by

$$\vec{\alpha}_{avg} = \frac{\vec{\omega_2} - \vec{\omega_1}}{t_2 - t_1} = \frac{\Delta \vec{\omega}}{\Delta t}$$

- Instantaneous angular acceleration $\vec{\alpha}_{inst} = \frac{d\vec{\omega}}{dt}$
- It is a vector quantity & is represented along the axis of rotation according to right hand thumb rule.
- Dimensional formula - $[M^0 L^0 T^{-2}]$
- S.I. unit - rad/s^2.

RELATION BETWEEN LINEAR ACCELERATION & ANGULAR ACCELERATION -:

- Let ω_1 & ω_2 be the angular velocities of the body at two instants of time t_1 & t_2. Angular acceleration α is given by

$$\alpha = \frac{\omega_2 - \omega_1}{t_2 - t_1}$$

- But
$$\omega_1 = \frac{v_1}{r}$$

$$\omega_2 = \frac{v_2}{r}$$

$$\alpha = \frac{\dfrac{v_2}{r} - \dfrac{v_1}{r}}{t_2 - t_1}$$

$$\Rightarrow \alpha = \frac{\dfrac{v_2 - v_1}{r}}{t_2 - t_1}$$

$$\Rightarrow \alpha = \frac{1}{r} \times \frac{v_2 - v_1}{t_2 - t_1}$$

$$\Rightarrow \alpha = \frac{a}{r} \qquad \left(\because a = \frac{v_2 - v_1}{t_2 - t_1} \right)$$

$$\Rightarrow \boxed{a = r\alpha}$$

$$\boxed{linear\ acceleration = radius\ \times angular\ acceleration}$$

- **In vector form**

    We know that $\qquad \vec{v} = \vec{\omega} \times \vec{r}$

    Differentiating both sides with respect to 't'

    $$\frac{d\vec{v}}{dt} = \frac{d}{dt}(\vec{\omega} \times \vec{r})$$

    $$\Rightarrow \frac{d\vec{v}}{dt} = \left(\frac{d\vec{\omega}}{dt} \times \vec{r} \right) + \left(\vec{\omega} \times \frac{d\vec{r}}{dt} \right)$$

    $$\Rightarrow \boxed{\vec{a} = (\vec{\alpha} \times \vec{r}) + (\vec{\omega} \times \vec{v})} \quad \left[\because \vec{\alpha} = \frac{d\vec{\omega}}{dt}\ \&\ \vec{v} = \frac{d\vec{r}}{dt} \right]$$

- $(\vec{\alpha} \times \vec{r})$ is directed along the tangent & $(\vec{\omega} \times \vec{v})$ is directed along the radius towards the centre.
- Therefore $(\vec{\alpha} \times \vec{r})$ is called as the *tangential component* & $(\vec{\omega} \times \vec{v})$ is called as the *radial component* .

So
$$\boxed{\vec{a} = \vec{a}_T + \vec{a}_r}$$

PROJECTILE -:

- A body projected into the space & is no longer being propelled by fuel is called a projectile.
- EXAMPLES -:

    A bullet fired from a gun

    A cricket ball thrown into space

    A bomb dropped from an aeroplane.

PROJECTILE PROJECTED IN VERTICALLY UPWARD DIRECTION -:

- Suppose a body is projected from the point A with an initial velocity 'u' in vertically upward direction. Its velocity decreases gradually (since g retards the motion) till it becomes 0 at B.

MAXIMUM HEIGHT -:

- It is the maximum distance travelled by the projectile in the vertical direction.

Here initial velocity = u
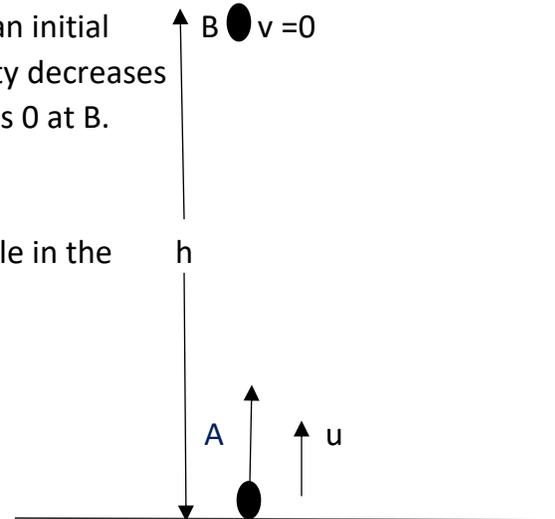
    Final velocity, v = 0

    Acceleration = -g

    Distance travelled = h

    Now using the relation

$$v^2 - u^2 = 2as$$

We can write

$$0 - u^2 = 2(-g)h$$

$$\Rightarrow u^2 = 2gh$$

$$\Rightarrow \boxed{h = \frac{u^2}{2g}} ------(1)$$

TIME OF ASCENT -:

- It is the time taken by the projectile to rise to the highest point.

    Here initial velocity = u

        Final velocity, v = 0

        Acceleration = -g

        Time = t

    Now using the relation

$$v = u + at$$

We can write

$$0 = u + (-g)t$$

$$\Rightarrow u = gt$$

$$\Rightarrow \boxed{t = \frac{u}{g}} -----(2)$$

TIME OF DESCENT -:

- It is the time taken by the projectile to come down from the highest point to the point from which it was projected.

Here initial velocity (u) = 0

Acceleration = g

Distance travelled = h

Time = t

Now using the relation

$$s = ut + \frac{1}{2}at^2$$

We can write

$$h = (0)t + \frac{1}{2}gt^2$$

$$\Rightarrow h = \frac{1}{2}gt^2$$

$$\Rightarrow t^2 = \frac{2h}{g}$$

Putting the value of 'h' from equation (1), we can write

$$\Rightarrow t^2 = \frac{2 \times \frac{u^2}{2g}}{g}$$

$$\Rightarrow t^2 = \frac{u^2}{g^2}$$

$$\Rightarrow \boxed{t = \frac{u}{g}} \; - - - - - - (3)$$

TIME OF FLIGHT -:

- It is the time taken by projectile to come back to the same level from which it is projected.

$$time \ of \ flight = time \ of \ ascent + time \ of \ descent$$

$$T = \frac{u}{g} + \frac{u}{g}$$

$$\Rightarrow \boxed{T = \frac{2u}{g}} \; - - - - - - - (4)$$

VELOCITY ON REACHING THE GROUND -:

Consider the motion of the body from B to A.

Here initial velocity (u) = 0

Final velocity - v

Acceleration = g

Distance travelled = h

Now using the relation

$$v^2 - u^2 = 2as$$
$$\Rightarrow v^2 - 0 = 2gh$$
$$\Rightarrow v^2 = 2gh$$
$$\Rightarrow v^2 = 2g \times \frac{u^2}{2g}$$
$$\Rightarrow v^2 = u^2$$
$$\Rightarrow v = \pm u$$

But we take $v = -u$

- Negative sign indicates that the velocity of the body on reaching the ground is in downward direction.
- Therefore velocity of the body on reaching the ground is same as the velocity with which the body was projected.

HORINZONTAL RANGE -:

- It is the distance travelled by the projectile in horizontal direction.
- In this case there is no component of velocity along horizontal direction. So it travels no distance in the horizontal direction.

Therefore horizontal range = 0

PROJECTILE FIRED HORIZONTALLY -:
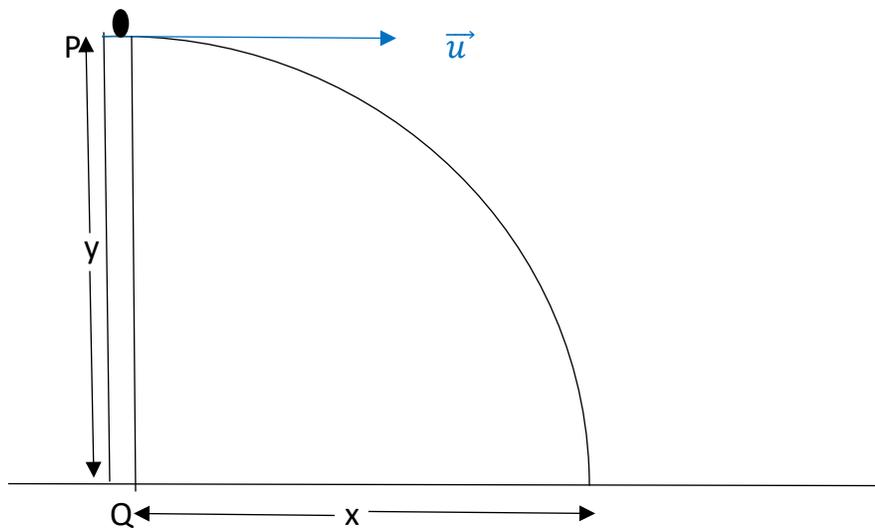
- Let us consider a projectile is projected with a velocity 'u' in horizontal direction from a height 'h' above the ground. As it moves through the space, it experiences two velocities:

1. A velocity 'u' along horizontal direction which remains constant throughout the motion of the body because acceleration due to gravity has no component along horizontal direction.

2. A velocity along vertical direction which is non-uniform. Its initial value is zero.

EQUATION OF TRAJECTORY -:

- It is an equation connecting the horizontal & vertical distances travelled by the projectile.

Horizontal equation of motion of the projectile is given by

$$x = ut + 0 \quad \left(using \ the \ relation \ s = ut + \frac{1}{2}at^2, acceleration(a) = 0\right)$$

$$\Rightarrow \boxed{x = ut} \text{------------- (1)}$$

Vertical equation of motion of the projectile is given by

$$y = (0)t + \frac{1}{2}gt^2 \quad \left(using \ the \ relation \ s = ut + \frac{1}{2}at^2, initial \ velocity(u) = 0\right)$$

$$\Rightarrow y = \frac{1}{2}gt^2$$

$$\Rightarrow y = \frac{1}{2}g \times \frac{x^2}{u^2} \quad \left(using \ equ. \ 1, t = \frac{x}{u}\right)$$

$$\Rightarrow x^2 = \frac{2u^2}{g}y$$

$$\Rightarrow \boxed{x^2 = k\,y} ----(2) \qquad \left[k = \frac{2u^2}{g}\right]$$

- This is the equation of a **parabola symmetric about y-axis**. Hence the path of the projectile is **parabolic**.

TIME OF DESCENT -:

- It is the time taken by the projectile to come down to the surface of earth. Considering the motion of the projectile along the vertical direction only.

Here initial velocity (u) = 0

    Acceleration = g

    Distance travelled = h

    Time = t

Now using the relation

$$s = ut + \frac{1}{2}at^2$$

We can write

$$h = (0)t + \frac{1}{2}gt^2$$

$$\Rightarrow h = \frac{1}{2}gt^2$$

$$\Rightarrow t^2 = \frac{2h}{g}$$

$$\Rightarrow \boxed{t = \sqrt{\frac{2h}{g}}} \quad \text{-------------(3)}$$
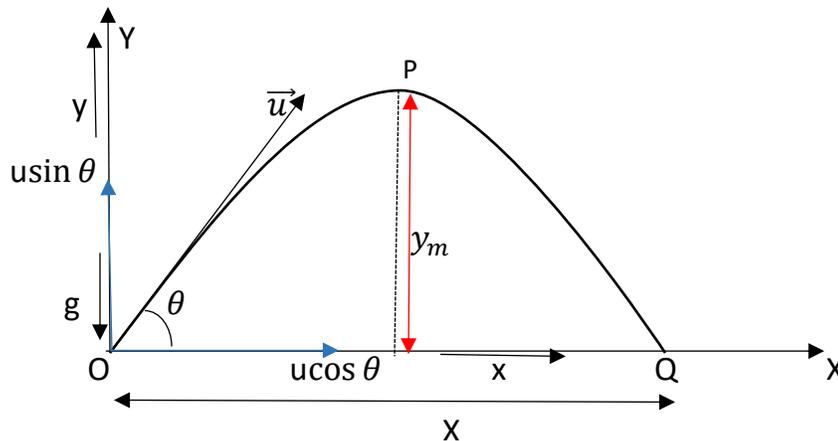
HORINZONTAL RANGE -:

- It is the distance travelled by the projectile in horizontal direction.
- The horizontal equation of motion of the projectile is given by

$$X = u \times t \quad \text{[t is the time of descent]}$$

$$\Rightarrow \boxed{X = u \times \sqrt{\frac{2h}{g}}} \quad \text{------------------(4)} \qquad \text{[using equ- (3)]}$$

This is the expression for horizontal range.

PROJECTILE FIRED AT AN ANGLE θ WITH THE HORIZONTAL -:

- Consider a projectile is fired with a velocity $\vec{u}$ by making an angle θ with the horizontal. The projectile rises to the highest point P & then comes back to the point Q which is on the label of projection.
- Resolving $\vec{u}$ into two rectangular components.

    (1) $u\cos\theta$ along horizontal direction which is uniform throughout the motion of the projectile.

    (2) $u\sin\theta$ along vertical direction which is non-uniform.

EQUATION OF TRAJECTORY -:

- Horizontal equation of motion of the projectile is given by

$$x = velocity \times time$$

$$x = (ucos\theta).t \text{ _____ } (1) \quad [using\ the\ relation\ s = ut + \frac{1}{2}at^2]$$

- Vertical equation of motion of the projectile is given by

$$y = (u\sin\theta)t - \frac{1}{2}gt^2$$

$$\Rightarrow y = (u\sin\theta) \times \frac{x}{ucos\theta} - \frac{1}{2}g\left(\frac{x}{ucos\theta}\right)^2 \quad [\text{from equ.} (1)]$$

$$\Rightarrow \boxed{y = x\tan\theta - \frac{gx^2}{2u^2cos^2\theta}} \text{ _____ } (2)$$

- This is the equation of a parabola which is symmetric about a line parallel to Y-axis. Hence the path followed by a projectile is parabolic in nature.

MAXIMUM HEIGHT -:

- It is the maximum distance travelled by the projectile in the vertical direction.
- Let us consider the motion of the projectile in vertical direction only.

    At O, initial vertical velocity = $u\sin\theta$

    At P, final vertical velocity = 0

    Acceleration = -g

    Vertical distance travelled = y_m

Using the relation $\qquad v^2 - u^2 = 2as$

We can write $\qquad 0 - u^2 sin^2\theta = 2(-g)y_m$

$$\Rightarrow \boxed{y_m = \dfrac{u^2 sin^2\theta}{2g}} \qquad \text{_____ (3)}$$

TIME OF ASCENT -:

- It is the time taken by the projectile to rise to the highest point.
- Again considering the motion of the projectile in vertical direction only.

At O, initial vertical velocity = u sin θ

At P, final vertical velocity = 0

Acceleration = -g

Time $= t_1$

Now using the relation

$$v = u + at$$

We can write $\qquad 0 = u \sin\theta + (-g)t_1$

$$\Rightarrow u \sin\theta = gt_1$$

$$\Rightarrow \boxed{t_1 = \dfrac{u \sin\theta}{g}} \qquad \text{_____ (4)}$$

TIME OF DESCENT -:

- It is the time taken by the projectile to come down from the highest point to the surface of earth.

At P, initial vertical velocity = 0

At Q, final vertical velocity = u sin θ

Acceleration = g

Time $= t_2$

Now using the relation

$$v = u + at$$

We can write $\qquad$ u sin $\theta = 0 + gt_2$

$$\Rightarrow u \sin\theta = gt_2$$

$$\Rightarrow \boxed{t_2 = \dfrac{u \sin\theta}{g}} \qquad \text{_____ (4)}$$

TIME OF FLIGHT -:

- It is the time taken by projectile to come back to the same level from which it was projected.

$$time\ of\ flight = time\ of\ ascent + time\ of\ descent$$

$$T = t_1 + t_2$$

$$\Rightarrow\ T = \frac{u\sin\theta}{g} + \frac{u\sin\theta}{g}$$

$$\Rightarrow\ \boxed{T = \frac{2u\sin\theta}{g}} \underline{\hspace{2cm}} (5)$$

HORINZONTAL RANGE -:

- It is the distance travelled by the projectile in the horizontal direction.

Now using the relation

$$s = ut + \frac{1}{2}at^2$$

$$X = horizontal\ velocity \times total\ time\ of\ flight$$

$$\Rightarrow X = u\cos\theta \times \frac{2u\sin\theta}{g}$$

$$\Rightarrow X = \frac{2u\cos\theta.\,u\sin\theta}{g}$$

$$\Rightarrow X = \frac{u^2(2\cos\theta.\,u\sin\theta)}{g}$$

$$\Rightarrow\ \boxed{X = \frac{u^2\sin 2\theta}{g}} \underline{\hspace{2cm}} (6)$$

CONDITION FOR MAXIMUM HORIZONTAL RANGE -:

- From equ-(6) it is cleared that horizontal range depends upon the velocity of projection (u) & angle of projection (θ).
- For a fixed value of u horizontal range can be changed by changing the value of θ.
- Horizontal range will be maximum if $\sin 2\theta$ is maximum i.e. 1.

$$\sin 2\theta = 1 = \sin 90°$$

$$\Rightarrow 2\theta = 90°$$

$$\Rightarrow \boxed{\boldsymbol{\theta = 45°}}$$

- This is the condition for maximum horizontal range.
- The maximum value of horizontal range is given by

$$\boxed{X_m = \frac{u^2}{g}} \rule{3cm}{0.4pt} \text{(7)}$$

Two angles of projection for same horizontal range

We know that horizontal range

$$X = \frac{u^2 \sin 2\theta}{g}$$

$$= \frac{u^2 \sin(180° - 2\theta)}{g}$$

$$= \frac{u^2 \sin 2(90° - \theta)}{g}$$

$$\boxed{X = \frac{u^2 \sin 2\theta'}{g}} \rule{2cm}{0.4pt} \text{(8)} \quad \left[\theta' = 90° - \theta\right]$$

- In equation (6) X is the horizontal range & θ is the angle of projection.
- In equation (8) X is the horizontal range but $\theta' = 90° - \theta$ is the angle of projection.
- Therefore for **two angles of projection** θ & $\theta' = 90° - \theta$**, the projectile will cover the same horizontal range.** These two angles θ & $90° - \theta$ are called as complementary angles because their sum is $90°$.

UNIT-4

WORK & FRICTION

WORK

- Work is said to be done by a force acting on the body, provided the body is displaced actually in any direction except in a direction perpendicular to the direction of force.
- When a constant force '$\vec{F}$' acting on a body produces a displacement '$\vec{s}$' in the body, then the work (W) done by the force is the **dot product** of force ($\vec{F}$) & displacement ($\vec{S}$)

$$W = \vec{F}.\vec{s}$$

$$\boxed{W = Fs\,\cos\theta}$$ 'θ' is the angle between $\vec{F}$ & $\vec{s}$.

- Work is a scalar quantity.
- It can be positive/negative/zero.
- S.I. unit is N.m = joule (J)
- C.G.S. unit is dyne.cm = erg
- Dimension of work is $[M^1 L^2 T^{-2}]$

FRICTION

- Force of friction is the force that develops at the surfaces of contact of two bodies & opposes their relative motion.
- This force acts tangentially to the interface of two bodies.

NORMAL REACTION

- Let us consider a block of weight 'mg' lying on a horizontal surface.
- When the body presses against a surface, the surface deforms even if it appears to be rigid.
- The deformed surface pushes the body with a normal force 'R' i.e. perpendicular to the surface.
- This is called normal reaction. It balances 'mg'.

NORMAL REACTION

FORCE OF FRICTION   **APPLIED FORCE**

WEIGHT
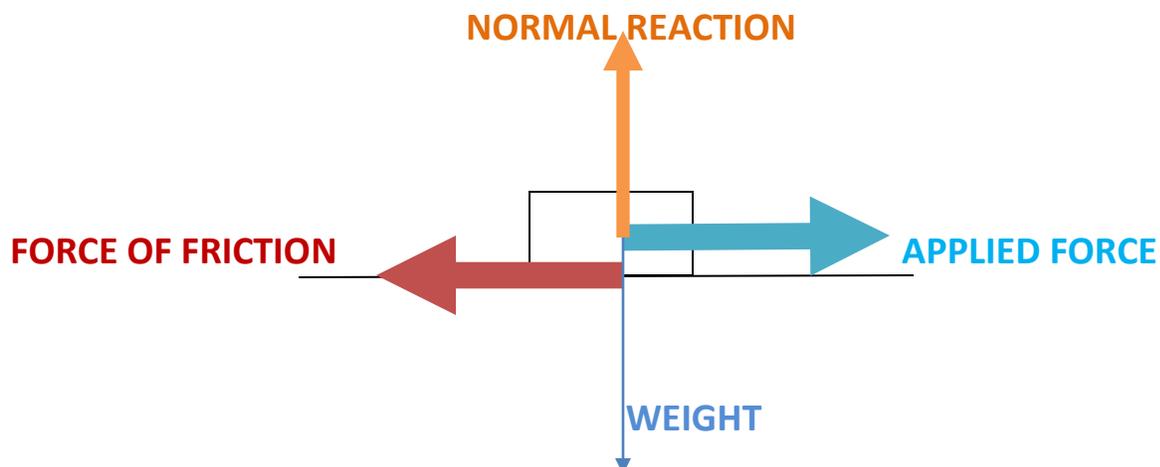
TYPES OF FRICTION

STATIC FRICTION, LIMITING FRICTION & KINETIC FRICTION

- Suppose a small force 'P' is applied on a block. The force of friction 'F' opposes the motion.
- So long as the block does not move

$$F = P$$

This means as we increase 'P' friction 'F' also increases, remaining equal to 'P' always.

STATIC FRICTION

- It is the opposing force that comes into play between two bodies in contact, when one body tends to move over the surface of another but the actual motion has yet not started.

LIMITING FRICTION

- It is the maximum opposing force that comes into play between two bodies in contact, when one body is just at the verge of moving over the surface of another.

DYNAMIC/KINETIC FRICTION

- It is the opposing force that comes into play between two bodies in contact, when one body is actually moving over the surface of another body.
- It is of 2 types.

1. SLIDING FRICTION

- It is the opposing force that comes into play between two surfaces in contact when one body is actually sliding over the surface of another body.
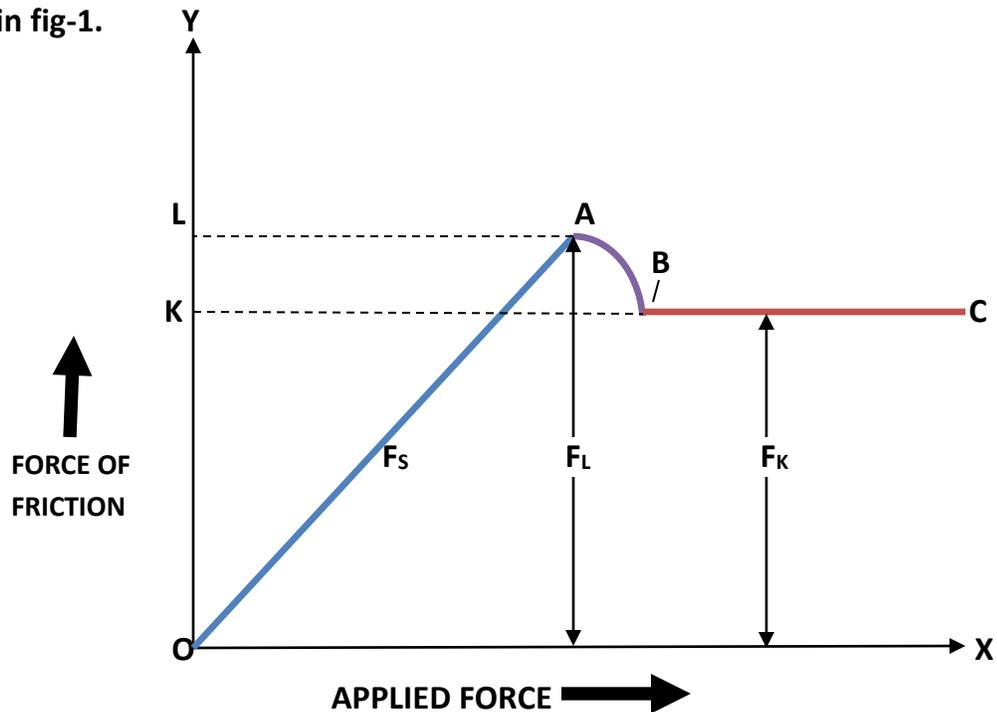
2. ROLLING FRICTION

- It is the opposing force that comes into play between two surfaces in contact when one body is actually rolling over the surface of another body.

- ## FLUID FRICTION

Fluid friction is the opposing force which comes into play when a body moves through a fluid.

If we plot a graph between applied force & the force of friction, we get a curve of the type shown in fig-1.

- The part OA of the curve represents static friction, F_s which goes on increasing with applied force.
- At A, the static friction is maximum. This represents the limiting friction (F_L = OL).
- Beyond A, the force of friction is seen decrease slightly.
- The portion BC of the curve, represents the kinetic friction (F_k = OK)
- Kinetic friction is always slightly less than the limiting friction.
- This is because, once the motion started actually, inertia of rest has been overcome. Also, when motion has actually started, irregularities of one surface have little time to get locked again into the irregularities of the other surface.

LAWS OF LIMITING FRICTION

1. The direction of force of friction is always opposite to the direction of motion.

2. The force of limiting friction depends upon the nature & state of polish of the surfaces in contact & act tangentially to the interface between the two surfaces.

3. The magnitude of limiting friction 'F' is directly proportional to the magnitude of normal reaction 'R' between the two surfaces in contact i.e. $F \alpha R$

4. The magnitude of limiting friction between two surfaces is independent of the area & shape of the surfaces in contact so long as the normal reaction remains the same.

COEFFICIENT OF FRICTION-

- According to the the law of limiting friction

$$F \, \alpha \, R$$
$$F = \mu \, R$$

Where μ is the constant of proportionality & is called the coefficient of limiting friction between two surfaces in contact.

$$\boxed{\mu = \frac{F}{R}}$$

- Coefficient of friction of a pair of surfaces in contact is defined as the ratio between the force of limiting friction 'F' to the normal reaction 'R'
- The value of μ depends on the nature of surfaces in contact & material of surfaces in contact.
- It has no unit & no dimension.
- When a body is actually moving over the surface of another body, we replace 'F' by 'F$_k$' & μ by μ$_k$.

$$\boxed{\mu_k = \frac{F_k}{R}}$$

μ_k is the coefficient of kinetic friction.

- Since $F_k < F$, therefore μ_k is always less than μ.

METHODS OF REDUCING FRICTION

1. BY RUBBING & POLISHING

- Polishing makes the surface smoother. Therefore friction reduces.

2.BY LUBRICANTS

- Lubricants like oil, grease etc fill up the irregularities of the surfaces, making them smoother. Hence friction decreases.

3. BY PROPER SELECTION OF MATERIALS

- Tyres are made up rubbers. This because friction between concrete & rubber is much less than the friction between iron & concrete.

4. BY STREAMLINING

- Friction due to air is considerably reduced by streamelining the shape of the object.

5. BY CONVERTING SLIDING INTO ROLLING FRICTION

- Rolling friction is much smaller than sliding friction.
- Sliding friction can be converted into rolling friction by means of ball bearing system.

UNIT-5

GRAVITATION

GRAVITATION-:

- It is a name given to the force of attraction between any two bodies in the universe.
- It was discovered by Newton in the year 1665, when he saw an apple falling down the tree.

NEWTON'S LAW OF GRAVITATION-:

STATEMENT- "Everybody in this universe attracts every other body with a force. The magnitude of this force is directly proportional to the product of their masses & is inversely proportional the square of distance between them."
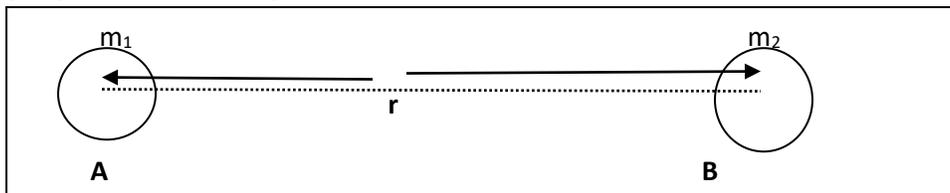
- Consider two bodies of masses m_1 & m_2. Let 'r' be the distance between their centers & 'F' be the force of attraction between them.
- Mathematically, $F \propto m_1 m_2$ & $F \propto \frac{1}{r^2}$
- So, $F \propto \frac{m_1 m_2}{r^2}$

$$F = G \frac{m_1 m_2}{r^2}$$

Where G is the constant of proportionality & is known as universal gravitational constant.

UNIVERSAL GRAVITATIONAL CONSTANT -:

From Newton's law of gravitation we know that

$$F = G \frac{m_1 m_2}{r^2}$$

$$G = \frac{F r^2}{m_1 m_2}$$

If $m_1 = m_2 = 1 unit$ & $r = 1 unit$
Then

$$G = F$$

- So, universal gravitational constant is defined as the force of attraction between two bodies of unit mass & separated by unit distance from each other.
  It is a scalar quantity.
- Its value is independent of nature & size of the bodies as well as the nature of the medium between the bodies.
- Its value is 6.67×10^{-11} Nm^2kg^{-2}.
- Its dimension is $[M^{-1} L^3 T^{-2}]$

GRAVITY -:

- It is a force of attraction exerted by earth towards its centre on a body lying on or near the surface of earth.
- It is a special case of gravitation & is also called earth's gravitational pull.
- The force with which a body is attracted towards the centre of earth is called its weight.

ACCELERATION DUE TO GRAVITY -:

- It is defined as the constant acceleration produced in a body when it falls freely under the effect of gravity alone.
- It is denoted by 'g'.
- S.I. unit is m/s^2 or N/kg.
- It is a vector quantity. Its direction is towards the centre of earth.
- Its value on the surface of earth is 9.8 m/s^2.
- Its dimension is $[M^0 L^1 T^{-2}]$

MASS	**WEIGHT**
1. It is the amount of matter contained in the body.	1. It is the force with which a body is attracted towards the centre of the earth.
2. It is a scalar quantity.	2. It is a vector quantity.
3. S.I. unit is kilogram(kg).	3. S.I. unit is newton(N).
4. Mass of a body is constant.	4. Weight of a body varies from place to place.
5. It is a fundamental physical quantity.	5. It is a derived physical quantity.
6. Mass of a body can never be equal to zero. .	6. Weight of a body is equal to zero at the centre of the earth.

RELATION BETWEEN g & G -:

Consider earth to be a spherical body of mass M & radius R with centre at O.

A body of mass m is placed at a point A on the surface of earth.

Let F be the gravitational force of attraction between the body & earth.
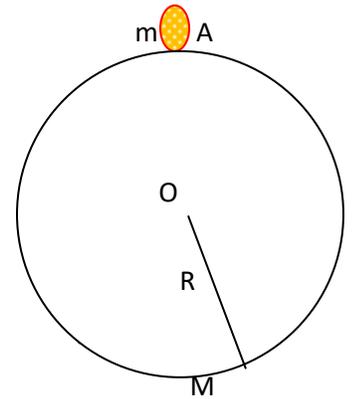
According to Newton's law of gravitation

$$F = G\frac{mM}{R^2} \quad \text{-----------} (1)$$

Where G is the universal gravitational constant

From earth's gravitational pull, $F = mg$ --------- (2)

Equating equ (1) & equ (2) $mg = G\frac{mM}{R^2}$

$$\boxed{g = \frac{GM}{R^2}}$$

VARIATION OF g WITH HEIGHT & DEPTH -:

1. EFFECT OF ALTITUDE-:

Consider earth to be a spherical body of mass M & radius R with centre at O.
Let g is acceleration due to gravity at a point A which is on the surface of earth.
We know that

$$g = \frac{GM}{R^2}$$

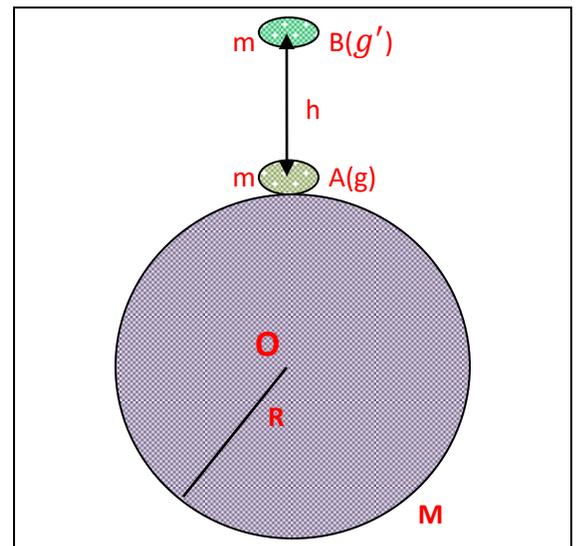Suppose g' is the acceleration due to gravity at a point B that is at a height h above the surface of earth.

$$g' = \frac{GM}{(R+h)^2}$$

When $h \ll R,$

$$\boxed{g' = g\left(1 - \frac{2h}{R}\right)}$$

So, $\boxed{g' < g}$

So the value of acceleration due to gravity decreases with increase in height.

2. EFFECT OF DEPTH -:

Consider earth to be a spherical body of mass M & radius R with centre at O. ρ is the uniform density of earth.

$$\rho = \frac{mass\ of\ earth}{volume\ of\ earth}$$

$$= \frac{M}{\frac{4}{3}\pi R^3}$$

$$M = \frac{4}{3}\pi\rho R^3$$

Let g is the acceleration due to gravity at a point A which is on the surface of earth.

We know that

$$g = \frac{GM}{R^2}$$

Suppose g' is the acceleration due to gravity at a point B that is at a depth d below the surface of earth.
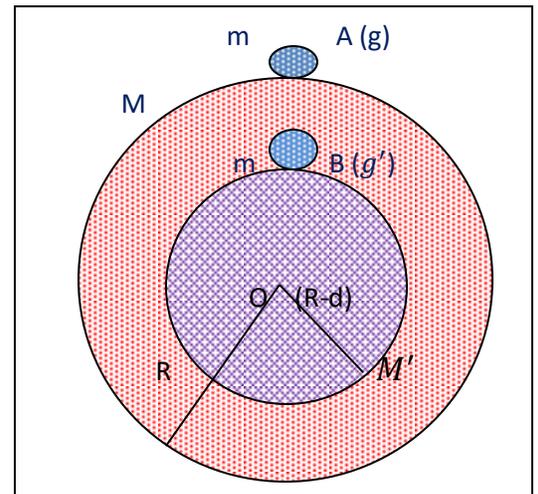
$$g' = \frac{GM'}{(R-d)^2}$$

Where, $\left[M' = \frac{4}{3}\pi\rho(R-d)^3\right]$

So,
$$\boxed{g' = g\left(1 - \frac{d}{R}\right)}$$
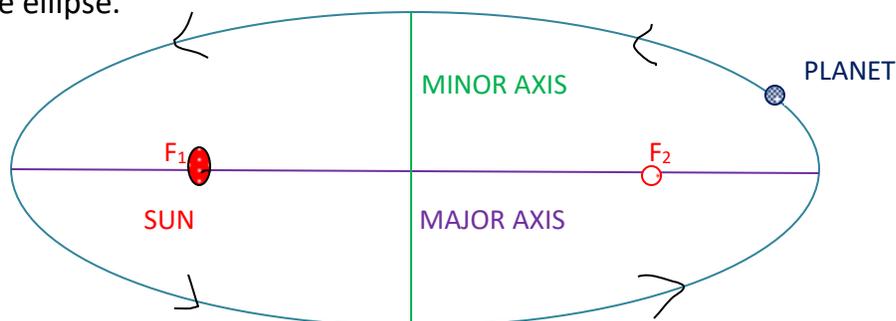
So,
$$\boxed{g' < g}$$

So the value of acceleration due to gravity decreases with increase in depth.

KEPLER'S LAWS OF PLANATORY MOTION -:

FIRST LAW (LAW OF ELLIPTICAL ORBIT) -:

Every planet revolves around sun in an elliptical orbit with sun situated at one of the foci of the ellipse.

SECOND LAW (LAW OF AREA) -:

A planet revolves around sun in such a way that the radius vector joining the planet to the sun sweeps out equal area in equal interval of time or the the areal velocity of the planet around sun is constant.
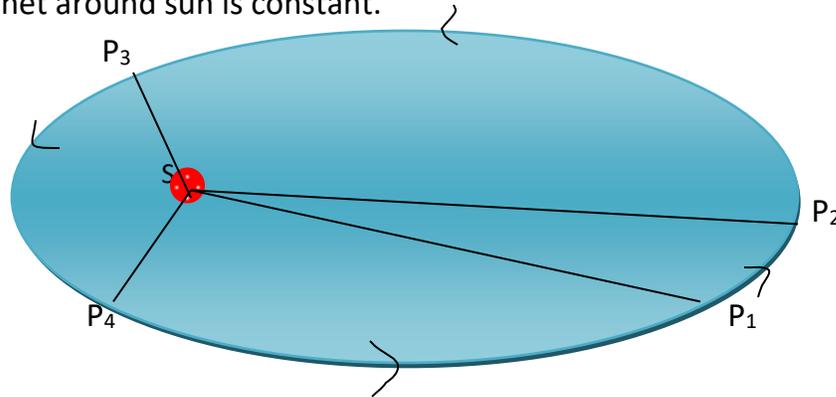
$$area\ of\ P_1SP_2 = area\ of\ P_3SP_4$$
$$(SP_1)(P_1P_2) = (SP_3)(P_3P_4)$$
$$since \quad SP_1 > SP_3$$
$$so \quad P_1P_2 < P_3P_4$$

$$or \qquad \boxed{\frac{P_1P_2}{t} < \frac{P_3P_4}{t}}$$

$$\frac{P_1P_2}{t} = linear\ velocity\ of\ the\ planet\ at\ P_1$$

$$\frac{P_3P_4}{t} = linear\ velocity\ of\ the\ planet\ at\ P_3$$

So the orbital velocity of a planet around sun is not constant.

- So the linear velocity of the planet closer to the sun is greater than the linear velocity of planet when away from the sun.

THIRD LAW (LAW OF TIMEPERIOD) -:

A planet revolves around sun in such a way that the square of time period of revolution of a planet around sun is directly proportional to the cube of semi major axis of the ellipse.

$$\boxed{T^2\ \alpha\ R^3}$$

where,

$$T = time\ taken\ by\ the\ planet\ to\ go\ once\ around\ the\ sun$$
$$R = semi\ major\ axis\ of\ the\ ellipse$$

- So a planet situated at larger distance takes longer time to complete one revolution around sun as compared to the time taken by the planet situated nearer to the sun.