Differential Equation

→ A eq^n with derivative is Known as Differential eq^n.

Following are the Example of differential eq^n.

(i) $\frac{dy}{dx} + 3y = x$

(2) $\frac{d^2y}{dx^2} + \frac{dy}{dx} = e^x$

(3) $\frac{d^3y}{dx^3} + \left(\frac{dy}{dx}\right)^5 = x$.

when an eq^n contain derivative with respect to a single independent Variable is Known as <u>Ordinary diff^n eq^n</u> (Above example), more than one independent variable is Known as partial Diff^n eq^n (exe- $\frac{\partial^2 z}{\partial y \partial x}$. etc)

Order of diff^n eq^n :-

The highest order derivative is consider the Order of diff^n eq^n.

→ Degree of diff^n eq^n :-

Power of highest order derivative is taken as the degree of diff^n eq^n.

Ex:1 $\left(\frac{d^2y}{dx^2}\right)^2 = 1 + \frac{dy}{dx}$

Order - 2

degree - 2

Ex:-3 $\frac{d^3y}{dx^3} + \left(\frac{dy}{dx}\right)^5 = x$

Here Order = 3

degree = 1

Ex:-2 $\frac{d^2y}{dx^2} + \frac{dy}{dx} = e^x$

Here Order = 2

degree = 1

* Solution of diffⁿ eqⁿ :-

A eqⁿ $y = f(x)$ or $f(x, y) = 0$ is called a Solⁿ of diffⁿ eqⁿ.

Ex:- $y = e^x$ is a Solⁿ of diffⁿ eqⁿ

Now if $y = e^x$

$$\frac{dy}{dx} = e^x$$

$$\frac{dy}{dx} = y$$ which is a solⁿ of diffⁿ eqⁿ.

* Formation of diffⁿ eqⁿ :- (How to make diffⁿ eqⁿ)

Ex:-1 $y = mx$

$$\frac{dy}{dx} = m$$

$$\frac{dy}{dx} = \frac{y}{x} \quad \left(\because y = mx \Rightarrow m = \frac{y}{x} \right)$$

which is diffⁿ eqⁿ.

Ex:-2 $x^2 + y^2 = a^2$

diffⁿ both side w.r.t x

$$\frac{d}{dx}(x^2 + y^2) = \frac{d}{dx}a^2$$

$$2x + 2y\frac{dy}{dx} = 0$$

$$2\left(x + y\frac{dy}{dx}\right) = 0$$

$$x + y\frac{dy}{dx} = 0$$

$$y\frac{dy}{dx} = -x$$

$$\frac{dy}{dx} = \frac{-x}{y} \Rightarrow \frac{dy}{dx} + \frac{x}{y} = 0$$ which is a diffⁿ eqⁿ of degree 1.

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Note Numbers of Parameter (constant) tends to degree of the diffn eqn.

Ex:-1 $y = A\sin x$

$\frac{dy}{dx} = A\cos x$

Here A is the parameter. we have to eliminate to this parameter.

$\Rightarrow \frac{dy}{dx} = \frac{y}{\sin x} \times \cos x$ $\left(\because y = A\sin x \Rightarrow A = \frac{y}{\cos x} \right)$

$\Rightarrow \frac{dy}{dx} = y\cot x$

which is a diffn eqn of order 1 (Because here 1 parament is there)

Ex:-2 $y = Ae^{x} + Be^{x}$

Here two parament. A & B is given

$\frac{dy}{dx} = Ae^{x} - Be^{-x}$

Again diffn w.r.t x

$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx} Ae^{x} - Be^{-x}$

$\frac{d^2y}{dx^2} = Ae^{x} + Be^{-x}$

$\frac{d^2y}{dx^2} = y$

which is a diffn eqn of order 2. (Because of two parament. Hence the order is 2)

Methods of Saving diff^n eq^n :-

(Separable method)

The eq^n (Type I)

$$\frac{dy}{dx} = f(x)$$

$$dy = f(x).dx$$

integrating both side

$$\int dy = \int f(x)dx$$

$$y = F'(x) + K.$$

ex:-1

Solve $\frac{dy}{dx} = x^2+2x+5$

Ans $dy = (x^2+2x+5)dx$

integrating both side

$$\int dy = \int (x^2+2x+5)dx$$

$$y = \int x^2dx + \int 2x + \int 5dx$$

$$y = \frac{x^3}{3} + 2\frac{x^2}{2} + 5x + K.$$

$$y = \frac{x^3}{3} + x^2 + 5x + K$$

Ex:-2

Solve $\frac{dy}{dx} = \tan y$

Ans: $\frac{dy}{\tan y} = dx$

$\cot y . dy = dx$

integrating both side

$\int \cot y . dy = \int dx$

$\log_e (\sin y) = x + c$

$\sin y = e^{(x+c)}$

$\sin y = e^{x} . e^{c}$

Ex:-4

Solve $\frac{dy}{dx} = \frac{2y}{x^2+1}$

Ans: $\frac{dy}{2y} = \frac{dx}{1+x^2}$

integrating on both side

$\int \frac{dy}{2y} = \int \frac{dx}{1+x^2}$

$= \frac{1}{2} \log_e y = \tan^{-1} x + K$

$\log_e y = 2 \tan^{-1} x + K$

$y = e^{2\tan^{-1} x + K}$ (Ans)

Type-II

$$\frac{d^2y}{dx^2} = h(x)$$

Let $\frac{dy}{dx} = p$

$$\Rightarrow \frac{d}{dx}\left(\frac{dy}{dx}\right) = h(x)$$

$$\frac{dp}{dx} = h(x)$$

$$dp = h(x)\,dh$$

Integrating both side

$$\int dp = \int h(x)$$

$$p = h'(x) + c$$

Again $p = \frac{dy}{dx}$

Now $\frac{dy}{dx} = h'(x)$

$$dy = \left(h'(x) + c\right)dx$$

$$y = h'(x) + cx + d$$

Ex:-1

Solve $\frac{d^2y}{dx^2} = 6x+2$

Ans: $P = \frac{dy}{dx}$

Now $\frac{d}{dx}\left(\frac{dy}{dx}\right) = 6x+2$

$= \frac{dP}{dx} = 6x+2$

$dP = (6x+2)dx$

integrating both side

$\int dP = \int (6x+2)dx$

$P = \int 6x\,dx + \int 2\,dx$

$P = 6.\frac{x^2}{2} + 2x + C$

$P = 3x^2 + 2x + C$

$\frac{dy}{dx} = 3x^2 + 2x + C$

$dy = (3x^2 + 2x + C)dx$

Again integrating

$\int dy = \int (3x^2 + 2x + C)dx$

$y = \int 3x^2 dx + \int 2x\,dx + \int C\,dx$

$\Rightarrow y = \frac{3x^3}{3} + \frac{2x^2}{2} + Cx + d$

$\Rightarrow y = x^3 + x^2 + Cx + d$ (Ans).

Particular ~~Sol~~ Sol^n :-

In particular Sol^n we put the particular value.

Ex:-

Solve $\frac{dy}{dx} = \cos x$ at $y = 2$ & $x = 0$

Ans Now

$$\frac{dy}{dx} = \cos x$$

$$dy = \cos . dx$$

Integrating both side

$$\int dy = \int \cos x . dx$$

$$y = \sin x + C$$

Put $x = 0$ & $y = 2$

$$2 = \sin 0 + C$$

$$\Rightarrow C = 2$$

Hence the particular Sol^n is

$$y = \sin x + 2$$

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Assignment Work.

① Solve $\frac{dy}{dx} = \sqrt{1-y^2}$

② Solve $\frac{dy}{dx} = y+2$

③ Find the General Solution

$$\frac{dy}{dx} \; x\cos x$$

④ Obtain the General soln

$$\frac{dy}{dx} = (x^2+1)(y^2+1)$$

⑤ Solve $\frac{d^2y}{dx^2} = 12x^2+2x$

⑥ Solve $\frac{d^2y}{dx^2} = \sec^2 x + \cos^2 x$

⑦ Form the diffn eqn and eliminating the constant term

$$y = A\sec x$$

⑧ Determine the degree and order of the diffn eqn

$$\left(\frac{dy}{dx}\right)^4 + y^5 = \frac{d^3y}{dx^3}$$

(9) Find the Particular solⁿ of

$$\frac{dy}{dx} = \cos x, \text{ given } y = 2, \text{ when } x = 0$$

(10) Find the Particular solⁿ of

$$\frac{dy}{dx} = \frac{1+y^2}{1+x^2}, \text{ given } y = \sqrt{3}, \text{ when } x = 1$$