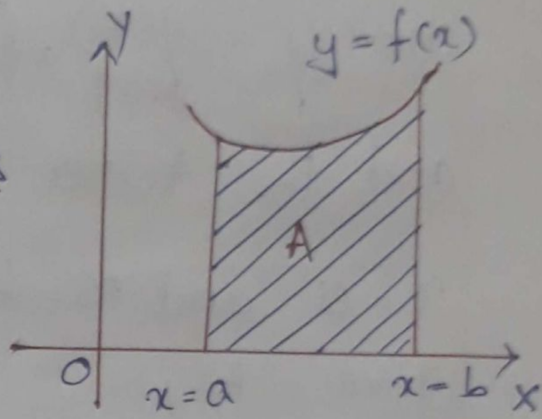
Area Bounded By The Curve

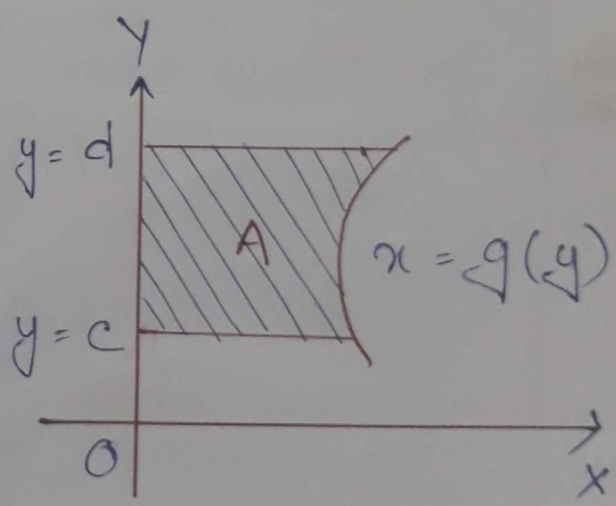
Let $f(x)$ be continuous in $[a,b]$

① Area bounded by the curve $y = f(x)$, the axis of x and the ordinates at $x = a$ and $x = b$ is



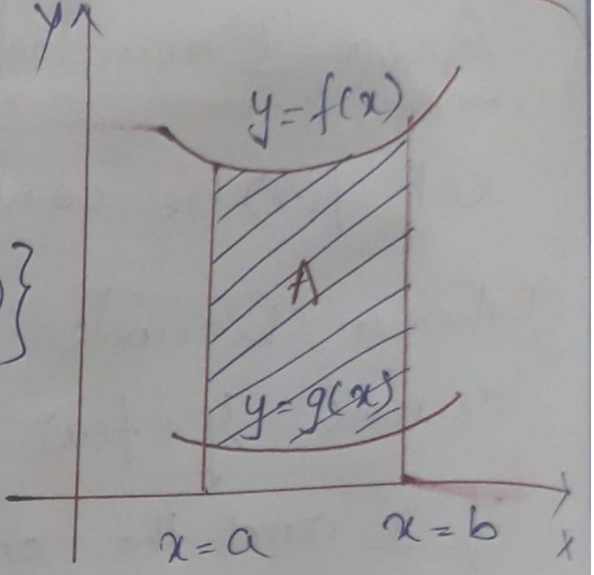
$$A = \int_a^b y \, dx = \int_a^b f(x) \, dx.$$

② Area bounded by the curve $x = g(y)$, the axis of y and the abscissa at $y = c$ and $y = d$ is



$$A = \int_c^d x \, dy = \int_c^d g(y) \, dy$$

(3) Area bounded by two curves $y = f(x)$ and $y = g(x)$, $\{f(x) > g(x)\}$ and the two ordinates $x = a$ and $x = b$ is give by.



$$A = \int_a^b \{f(x) - g(x)\}\, dx$$

① Q. Find the area bounded by $y = e^x$, $y = 0$, $x = 4$, $x = 2$.

A - Area $= \int_2^4 y\,dx = \int_2^4 e^x\,dx$

$$= \left[e^x\right]_2^4$$

$$= e^4 - e^2 \text{ square units.}$$

② Q. Find the area bounded by $xy = a^2$, $y = 0$, $x = \alpha$, $x = \beta$. $(\beta > \alpha > 0)$

A - ~~Are~~ Given $xy = a^2$

$$\Rightarrow y = \frac{a^2}{x}$$

Area $= \int_\alpha^\beta y\,dx = \int_\alpha^\beta \frac{a^2}{x}\,dx$

$$= a^2 \int_\alpha^\beta \frac{1}{x}\,dx$$

$$= a^2 \left[\log x\right]_\alpha^\beta$$

$$= a^2 \left[\log \beta - \log \alpha\right] = a^2 \log\left(\frac{\beta}{\alpha}\right) \text{ sq. units}$$

③ Q Find the area enclosed by $y = e^x$, $x = 0$, $y = 2$ & $y = 3$

A $\Rightarrow \log y = \log e^x$

$\Rightarrow \boxed{\log y = x}$

$$\text{Area} = \int_a^b x\,dy = \int_2^3 \log y\,dy$$

$=$ Let $u = \log y$, $v = 1$

$= \log y \cdot \int 1\,dy - \int\left(\frac{d}{dy}\log y \cdot \int 1\,dy\right)dy$

$= y \cdot \log y - \int \frac{1}{\cancel{y}} \cdot \cancel{y}\,dy$

$= y \cdot \log y - y + c$

$\therefore \int_2^3 \log y\,dy = \left[y\log y - y\right]_2^3$

$= (3\log 3 - 3) - (2\log 2 - 2)$

$= \log 3^3 - 3 - \log 2^2 + 2$

$= \log 27 - \log 4 - 1$

$= \log 27 - (\log 4 + \log e)$

$= \log\left(\frac{27}{4e}\right)$

$\left[\because \log e = 1\right.$ & $\log 4 + \log e = \log(4 \cdot e)$

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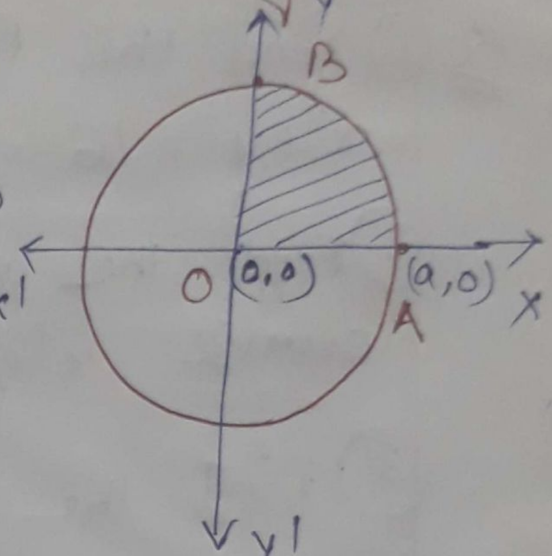
Q(4) Find the area of the circle $x^2+y^2=a^2$.

A- Given $x^2+y^2=a^2$

Here centre is at origin

and radius $= a$.



$y^2 = a^2 - x^2$

$\Rightarrow y = \sqrt{a^2 - x^2}$

Area of OAB $= \int_0^a y\,dx$

whole Area of the circle $= 4 \times \int_0^a y\,dx$

$= 4 \times \int_0^a \sqrt{a^2-x^2}\,dx$

$= 4 \times \left[\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}x/a\right]_0^a$

$= 4 \times \left[\left(a/2\sqrt{a^2-a^2} + \frac{a^2}{2}\sin^{-1}a/a\right) - \left(0 + \frac{a^2}{2}\sin^{-1}0\right)\right]$

$= 4\left(\frac{a^2}{2} \times \sin^{-1}1 - \frac{a^2}{2}\sin^{-1}0\right)$

$= 4 \times \left(\frac{a^2}{2} \times \pi/2 - 0\right)$

$= \cancel{4} \times \frac{\pi a^2}{\cancel{4}} = \pi a^2$ sq. units.

Assignment

Q: (1.) Find the total area of the circle $x^2+y^2=16$

(2) Find the area bounded by the curve $x^2+y^2=9$.

(3) Find the area bounded by the curve $xy=c^2$, the x-axis and $x=2$, $x=3$.