# Definite Integral

**Theorem** :- If $f(x)$ is continuous on the closed interval $[a, b]$ and $F(x)$ is any indefinite integral of $f(x)$, then

$$\boxed{\int_a^b f(x)\,dx = \left[F(x)\right]_a^b = F(b) - F(a)}$$

Here 'a' and 'b' are respectively known as lower and upper limit of the integral.

→ The definite integral doesnot depends upon the constant of integration because

if $\int f(x)\,dx = F(x) + c$

then
$$\int_a^b f(x)\,dx = \left[F(x) + c\right]_a^b$$
$$= \{F(b) + c\} - \{F(a) + c\}$$
$$= F(b) + \cancel{c} - F(a) - \cancel{c}$$
$$= F(b) - F(a)$$

→ When the variable in the definite integral $\int_a^b f(x)\,dx$ is changed from $x$ to $t$, the new limits are the values of $t$ which correspond to the values of $a$ & $b$ of $x$.

# Properties of Definite Integrals

1) $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt = \int_a^b f(y)\,dy$

2) $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

3) $\int_a^a f(x)\,dx = 0$

4) $\int_a^b k f(x)\,dx = k\int_a^b f(x)\,dx$

5) $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

where $a < c < b$

6) $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

7) $\int_{-a}^a f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx, & \text{if } f(-x) = f(x) \text{ i.e. even f}^n \\ 0, & \text{if } f(-x) = -f(x) \text{ i.e. odd f}^n \end{cases}$

8) $\int_0^{2a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx, & \text{if } f(2a-x) = f(x) \\ 0, & \text{if } f(2a-x) = -f(x) \end{cases}$

9) $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

1) Q. Evaluate $\int_1^2 x^3 dx$

A- $\int_1^2 x^3 dx$

$= \left[\frac{x^4}{4}\right]_1^2$

$= \frac{1}{4}\left[2^4 - 1^4\right]$

$= \frac{1}{4}(16-1)$

$= \frac{15}{4}$

2) Q. Evaluate $\int_0^{\pi/2} \cos x \, dx$

A- $\int_0^{\pi/2} \cos x \, dx$

$= \left[\sin x\right]_0^{\pi/2}$

$= \sin \pi/2 - \sin 0$

$= 1 - 0$

$= 1$

3) Q. Evaluate $\int_0^1 \frac{1}{\sqrt{1-x^2}} dx$

A- $\int_0^1 \frac{1}{\sqrt{1-x^2}} dx$

$= \left[\sin^{-1} x\right]_0^1 = \sin^{-1} 1 - \sin^{-1} 0$

$= \pi/2 - 0$

$= \pi/2$

(4) **Q** Evaluate $\int_1^2 \log x \, dx$

A- We first find the indefinite integral i.e $\int \log x \, dx$.

(Applying By parts)

Here $u = \log x$, $v = 1$

$= \log x \cdot \int 1 \, dx - \int \left( \frac{d}{dx} \log x \cdot \int 1 \cdot dx \right) dx$

$= \log x \cdot x - \int \left( \frac{1}{x} \cdot x \right) dx$

$= x \log x - \int 1 \, dx$

$= x \log x - x + c$

$\therefore \int_1^2 \log x \, dx = \left[ x \log x - x \right]_1^2$

$= (2 \log 2 - 2) - (1 \log 1 - 1)$

$= 2 \log 2 - 2 - (0 - 1)$ $[\because \log 1 = 0]$

$= 2 \log 2 - 2 + 1$

$= 2 \log 2 - 1$

mod $f^n$

(5) Q. Evaluate $\int_{-2}^{3} |x|\, dx$

A- $\int_{-2}^{3} |x|\, dx$

Applying property

$= \int_{-2}^{0} |x|\, dx + \int_{0}^{3} |x|\, dx$ $\left[\because \int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx \text{ where } a<c<b\right]$

$= \int_{-2}^{0} -x\, dx + \int_{0}^{3} x\, dx$

$= \int_{0}^{-2} x\, dx + \int_{0}^{3} x\, dx$

$= \left[\frac{x^2}{2}\right]_0^{-2} + \left[\frac{x^2}{2}\right]_0^{3}$

$= \left[\frac{(-2)^2}{2} - 0\right] + \left[\frac{(3)^2}{2} - 0\right]$

$= \frac{4}{2} + 9/2$

$= 13/2$

greatest integer $f^n$

(6) Q. Evaluate $\int_{2}^{5} [x]\, dx$

A- $\int_{2}^{5} [x]\, dx$

$= \int_{2}^{3} [x]\, dx + \int_{3}^{4} [x]\, dx + \int_{4}^{5} [x]\, dx$ $[\because 2<3<4<5]$

$$= \int_2^3 2\,dx + \int_3^4 3\,dx + \int_4^5 4\,dx$$

$$= \cancel{2(3-2) + 3(4-3)}$$

$$= 2[x]_2^3 + 3[x]_3^4 + 4[x]_4^5$$

$$= 2(3-2) + 3(4-3) + 4(5-4)$$

$$= 2 + 3 + 4$$

$$= 9$$

(7) Q Evaluate $\int_{-3}^{3} (x + |x|)\,dx$

A-

$$= \int_{-3}^{0} (x + |x|)\,dx + \int_0^3 (x + |x|)\,dx$$

$$= \int_{-3}^{0} (x - x)\,dx + \int_0^3 (x + x)\,dx$$

$$= 0 + 2\int_0^3 x\,dx$$

$$= 2 \times \left[\frac{x^2}{2}\right]_0^3$$

$$= 2 \times \left(\frac{3^2}{2} - \frac{0^2}{2}\right)$$

$$= \cancel{2} \times \frac{9}{\cancel{2}}$$

$$= 9$$

imp

(8) Q. Evaluate $\int_0^{\pi/2} \log \tan x \, dx$

A- Let $I = \int_0^{\pi/2} \log \tan x \, dx$ ....... (I)

$I = \int_0^{\pi/2} \log \tan(\pi/2 - x) \, dx$ [Applying property (6)]

$= \int_0^{\pi/2} \log \cot x \, dx$ ...... (ii)

Adding eqn (1) & (2)

$I + I = \int_0^{\pi/2} \log \tan x \, dx + \int_0^{\pi/2} \log \cot x \, dx$

$= \int_0^{\pi/2} \log(\tan x \cdot \cot x) \, dx$

$= \int_0^{\pi/2} \log 1 \, dx$

$= \int_0^{\pi/2} 0 \, dx$

$\Rightarrow 2I = 0$

$\Rightarrow I = 0$

gmp

(9) Q. Evaluate $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$

A - Let $I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx \longrightarrow$ (i)

$\Rightarrow I = \int_0^{\pi/2} \frac{\sqrt{\sin(\pi/2 - x)}}{\sqrt{\sin(\pi/2-x)}+\sqrt{\cos(\pi/2-x)}}\,dx$ $\left[\because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right.$

$\Rightarrow I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx \longrightarrow$ (ii)

Adding equn (i) & (ii), we get

$I + I = \int_0^{\pi/2} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$

$\Rightarrow 2I = \int_0^{\pi/2} 1\,dx$

$= [x]_0^{\pi/2}$

$= (\pi/2 - 0)$

$\Rightarrow \boxed{I = \pi/4}$ (Ans)

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(10) Q. Evaluate $\int_0^{\pi/2} \frac{dx}{1+\tan x}$

A- $\int_0^{\pi/2} \frac{dx}{1+\frac{\sin x}{\cos x}}$

$= \int_0^{\pi/2} \frac{dx}{\left(\frac{\cos x+\sin x}{\cos x}\right)}$

$= \int_0^{\pi/2} \frac{\cos x}{(\cos x+\sin x)} dx$

Let $I = \int_0^{\pi/2} \frac{\cos x}{\cos x+\sin x} dx \longrightarrow$ (i)

$I = \int_0^{\pi/2} \frac{\cos(\pi/2 - x)}{\cos(\pi/2-x)+\sin(\pi/2-x)} dx$

$= \int_0^{\pi/2} \frac{\sin x}{\sin x+\cos x} dx \longrightarrow$ (ii)

Adding (i) & (ii) we get

$2I = \int_0^{\pi/2} \frac{\cos x+\sin x}{\sin x+\cos x} dx$

$= \int_0^{\pi/2} 1\, dx$

$\Rightarrow 2I = [x]_0^{\pi/2} = \pi/2$

$\Rightarrow \boxed{I = \pi/4}$ (Ans)

# Assignments

Q Evaluate

(1) $\int_2^4 x^2\,dx$

(2) $\int_0^{\pi/2} \sin x\,dx$

(3) $\int_0^1 \frac{dx}{e^x + e^{-x}}$

(4) $\int_1^2 \frac{2x}{1+x^2}\,dx$

(5) $\int_{-2}^3 [x]\,dx$

(6) $\int_{-3}^4 |x|\,dx$

(7) $\int_{-2}^2 ([x] + |x|)\,dx$

(8) $\int_0^{\pi/2} \frac{dx}{1+\cot x}$

(9) $\int_0^{\pi/2} \frac{\sqrt{\tan x}}{\sqrt{\tan x} + \sqrt{\cot x}}\,dx$

Imp

(10) Prove that

$$\int_0^{\pi/2} \log(\sin x)\,dx = -\frac{\pi}{2}\log 2$$