also when the powers of $\sin^m x \cdot \cos^n x$ is an odd positive integer (either m or n) the broke the odd power of $\sin^m x \cdot \cos^n x$ and then simplify.

Example:

$\int \sin^3 x \cdot \cos^3 x \, dx$ * Here both m & n are odd so both broke one power of them

$\Rightarrow \int \sin^2 x \cdot \sin x \cdot \cos^3 x \, dx$

$\Rightarrow \int (1-\cos^2 x) \cdot \sin x \cdot \cos^3 x \, dx$

Let $u = \cos x$

$\frac{du}{dx} = \frac{d}{dx}\cos x$

$\frac{du}{dx} = -\sin x$

$dx = -\frac{du}{\sin x}$

Now

$\Rightarrow \int (1-u^2) \cdot \cancel{\sin x} \cdot u^3 - \frac{du}{\cancel{\sin x}}$

$\Rightarrow -\int (u^3 - u^5)\, du$

$\Rightarrow -\int u^3\, du + \int u^5\, du$

$\Rightarrow -\frac{u^4}{4} + \frac{u^6}{6} + C$

$\Rightarrow -\frac{\sin^4 x}{4} + \frac{\sin^6 x}{6} + C$

2) $\int \sin^4 x \cdot \cos^3 x\, dx$

Here $\cos^3 x$ is the odd power its brok into $\cos^2 x \cdot \cos x$

$\Rightarrow \int \sin^4 x \cdot \cos^2 x \cdot \cos x\, dx$

$\Rightarrow \int \sin^4 x \cdot (1 - \sin^2 x) \cdot \cos x\, dx$

Let $u = \sin x$

$\frac{du}{dx} = \frac{d}{dx}\sin x$

$\frac{du}{dx} = \cos x$

$dx = \frac{du}{\cos x}$

Now

$\int u^4 (1-u^2) \cdot \cancel{\cos x} \times \frac{du}{\cancel{\cos x}}$

$\Rightarrow \int u^4 (1-u^2)\, du$

$\Rightarrow \int (u^4 - u^6)\, du$

$\Rightarrow \int u^4\, du - \int u^6\, du$

$\Rightarrow \frac{u^5}{5} - \frac{u^7}{7} + C$

put $u = \sin x$ then

$\frac{\sin 5x}{5} - \frac{\sin 7x}{7} + C$ (ans)

(iv) $\int \tan^6 \theta \, d\theta$

$\Rightarrow \int \tan^4\theta . \tan^2\theta \, d\theta$

$\Rightarrow \int \tan^4\theta \, (\sec^2\theta - 1) d\theta$

$\Rightarrow \int \tan^4\theta . \sec^2\theta d\theta \int \tan^4\theta \, d\theta$

Let $u = \tan\theta$

$\frac{du}{d\theta} = d/d\theta \tan\theta$

$\frac{du}{d\theta} = \sec^2\theta$

$du = \sec^2\theta . d\theta$

Now

$\Rightarrow \int u^4 du - \int \tan^4\theta \, d\theta$

$\Rightarrow \int u^4 du - \int \tan^2\theta . \tan^2\theta \, d\theta$

$\Rightarrow \int u^4 du - \int \tan^2\theta - (\sec^2\theta - 1) d\theta$

$\Rightarrow \frac{u^5}{5} - \int \tan^2\theta . \sec^2\theta d\theta - \int \tan^2\theta \, d\theta$

$\Rightarrow \frac{u^5}{5} - \int u^2 du + \int (\sec^2\theta - 1) \, d\theta$

$\Rightarrow \frac{u^5}{5} - \frac{u^3}{3} + \int \sec^2\theta d\theta - \int 1 . d\theta$

$\Rightarrow \frac{u^5}{5} - \frac{u^3}{3} + \tan\theta - \theta + C$