(6) $\int \frac{\cos x}{\sin x} dx$

$= \int \cot x \, dx$

$= \log(\sin x) + c$

Integration by Substitution:

Some time the given integral $\int f(x) dx$ can be reduce to one of the standard from by changing the independent variable x to t by substituting $x = \phi(t)$

Q Evalute:

$\int \tan x \, dx$

$= \int \sin x / \cos x \, dx$

Let $\cos x = t$

$\Rightarrow -\sin x \, dx = dt$

$\Rightarrow \sin x \, dx = -dt$

$\int \frac{-dt}{t}$

$= -\int \frac{1}{t} dt$

$= -\log t + c$

$= -\log(\cos x) + c$

(2) $\int (ax+b)^n dx$

Let $u = ax+b$

$\frac{du}{dx} = \frac{d}{dx}(ax+b)$

$\Rightarrow \frac{du}{dx} = \frac{d(ax)}{dx} + \frac{d(b)}{dx}$

$\Rightarrow \frac{du}{dx} = \frac{a\,d(x)}{dx} + 0$

$\Rightarrow \frac{du}{dx} = a$

$\Rightarrow dx = \frac{du}{a}$

Now

$\int u^n \frac{du}{a}$

$= \frac{1}{a} \int u^n dx$

$= \frac{1}{a} \left(\frac{u^{n+1}}{n+1} \right) + c$

$= \frac{1}{a} \left\{ \frac{(ax+b)^{n+1}}{n+1} \right\} + c$

Formula

$\rightarrow \int \cos(ax+b)\,dx = 1/a \sin(ax+b) + k$

$\rightarrow \int \sin(ax+b)\,dx = -1/a \cos(ax+b) + k$

$\rightarrow \int \sec^2(ax+b)\,dx = 1/a \tan(ax+b) + k$

$\rightarrow \int \operatorname{cosec}^2(ax+b)\,dx = -1/a \cot(ax+b) + k$

$\Rightarrow \int \sec(ax+b) \cdot \tan(ax+b) = 1/a \sec(ax+b) + k$

$\Rightarrow \int \operatorname{cosec}(ax+b) \cdot \cot(ax+b) = -1/a \operatorname{cosec}(ax+b)$

$\Rightarrow \int e^{ax+b}\,dx = 1/a\, e^{ax+b} + k$

$\Rightarrow \int \frac{dx}{\sqrt{1-(ax+b)^2}} = 1/a \sin^{-1}(ax+b)$

$\Rightarrow \int \frac{1}{1+(ax+b)^2}\,dx = 1/a \tan^{-1}(ax+b)$

$\Rightarrow \int \frac{dx}{ax+b} = \frac{1}{a} \log|ax+b| + k$

$\Rightarrow \int \cot x = \frac{1}{a} |\sin x| + k$

$\Rightarrow \int \tan x = \log|\sec x| + k$

$\Rightarrow \int \sec x\,dx = \log|\sec x + \tan x| + k$

$\Rightarrow \int \operatorname{cosec} x\,dx = \log|\operatorname{cosec} x - \cot x| + k$

Example

$$\int \frac{x^4 + 4x^3}{x^5 + 5x^4 + 7} dx$$

let

$u = x^5 + 5x^4 + 7$

$\frac{du}{dx} = \frac{d}{dx}(x^5 + 5x^4 + 7)$

$\frac{du}{dx} = 5x^4 + 4 \times 5x^3$

$\frac{du}{dx} = 5x^4 + 20x^3$

$\frac{du}{dx} = 5(x^4 + 4x^3)$

$\frac{du}{dx} = 5(x^4 + 4x^3)$

$dx = \frac{du}{5(x^4 + 4x^3)}$

Now

$\Rightarrow \int \frac{\cancel{x^4 + 4x^3}}{u} \times \frac{du}{5(\cancel{x^4 + 4x^3})}$

$\Rightarrow \frac{1}{5} \int \frac{du}{u}$

$\Rightarrow \frac{1}{5} \log|u| + c$

$\Rightarrow \frac{1}{5} \log|x^5 + 5x^4 + 7| + c$

2) $\int \sin^7 x . \cos x \, dx$

let

$u = \sin x$

$\frac{du}{dx} = \frac{d}{dx} \sin x$

$\frac{du}{dx} = \cos x$

$dx = \frac{du}{\cos x}$

now

$\int u^7 . \cos x . dx$

$\Rightarrow \int u^7 . \cancel{\cos x} \frac{du}{\cancel{\cos x}}$

$\Rightarrow \int u^7 . du$

$\Rightarrow \frac{u^{7+1}}{7+1} + c$

$\Rightarrow \frac{u^8}{8} + c \Rightarrow \frac{\sin^8 x}{8} + c$

3) $\int \frac{(\tan^{-1} x)^3}{1+x^2} dx$

$u = \tan^{-1} x$

$\frac{du}{dx} = \frac{d}{dx} \tan^{-1} x$

$\frac{du}{dx} = \frac{1}{1+x^2}$

$dx = (1+x^2) du$

Now

$\int \frac{u^3}{\cancel{1+x^2}} \cancel{(1+x^2)} du$

$\Rightarrow \int u^3 du \Rightarrow \frac{u^{3+1}}{3+1} + c \Rightarrow \frac{u^4}{4} + c$

$\Rightarrow \frac{(\tan^{-1} x)^4}{4} + c$

4) $\int 2e^{\tan^2 x} . \tan x . \sec^2 x \, dx$

$u = \tan^2 x$

$\frac{du}{dx} = d/dx \tan^2 x$

$\frac{du}{dx} = 2 \tan x . d/dx \tan x$

$\frac{du}{dx} = 2 \tan x . \sec^2 x$

$dx = \frac{du}{2 \tan x . \sec^2 x}$

Now

$\int 2e^u \cancel{\tan x . \sec^2 x} \frac{du}{\cancel{2 \tan x . \sec^2 x}}$

$= \int e^u \, du$

$= e^u + c$

$= e^{\tan^2 x} + c$

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